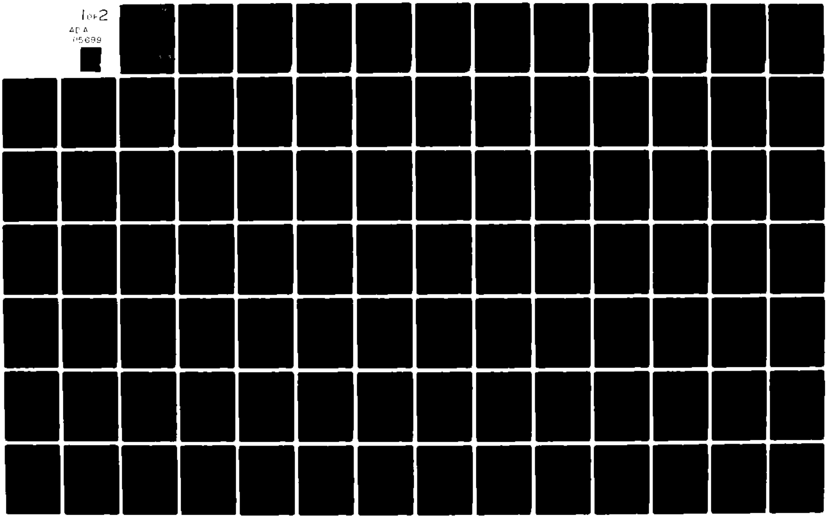
AD-A115 699 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOO--ETC F/G 15/7
AN AERIAL REFUELING OPTIMIZATION MODEL APPLIED TO STRATEGIC AIR--ETC(U)
MAR 82 T A LINDHOLM
UNCLASSIFIED AFIT/GST/OS/82M-7 NL

1 OF 2
AD A
115699



AFIT/GST/OS/82M-7

(1)

AD A115699

AN AERIAL REFUELING OPTIMIZATION MODEL APPLIED TO STRATEGIC AIRLIFT

THESIS

AFIT/GST/OS/82M-7 Tenny A. Lindholm
Capt USAF

Approved for public release; distribution unlimited

AFIT/GST/OS/82M-7

AN AERIAL REFUELING OPTIMIZATION MODEL APPLIED TO STRATEGIC AIRLIFT

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

Tenny A. Lindholm, B.S., M.B.A.
Captain USAF

Graduate Strategic and Tactical Sciences

March 1982

Preface

The Aerial Refueling Branch of the Airlift Management Division, Headquarters, Military Airlift Command (HQ MAC/DOOR), has, for some time, required an analytical method for optimizing the use of aerial refueling on strategic airlift missions. This study was conceived to satisfy this requirement. Up till now, MAC planners have dealt with the problem of how to efficiently employ the use of tankers either from past experience or by iteratively planning a mission to test various scenarios. This study provides a model that will determine optimal rendezvous points, fuel offloads, and tanker departure bases, using the total fuel consumed by both airlifter and tanker as the measure of effectiveness. Use of this model will insure optimal use of tanker resources, save considerable time during aerial refueling mission planning, and will result in a considerable fuel savings to the Air Force.

Special thanks goes to Major Gerald Armstrong, my thesis advisor, for initially providing a possible solution technique for this very complex problem and his continual guidance during the preparation of this report. I would also like to express my sincere appreciation to Professor Dan Reynolds, my reader, for his valuable comments, and

his wife, Phyllis, for her outstanding work in preparing the final copy of this report. LtCol Tom Baars and Capt Mark Kahley, both of HQ MAC/DOOR, also provided invaluable assistance in insuring that the aerial refueling optimization model was a valid one and that it could in fact be used operationally. Finally, I want to thank my wife, Lenita, for her faithful support and sacrifice during some busy and difficult times.

Contents

List of Figures

List of Tables

Abstract

With the advent of strategic airlift in-flight refueling, the role of strategic mobility in supporting our national policy has taken on new dimensions. Airlift of important military equipment to any part of the world is no longer dependent on enroute bases for refueling. Also, our response time to any contingency far from home can now be measured in terms of hours instead of days. This increased demand on our limited aerial refueling resources, plus the high cost and doubtful availability of jet fuel, require that any deployment using aerial refueling be done as efficiently as possible.

The main objective of this study is to define and develop an operationally usable optimization model that will, given airlifter route of flight, takeoff fuel and gross weight, enroute wind factors, tanker departure bases, and available enroute diversion bases, determine the aerial refueling points, fuel offload, and tanker departure bases to minimize total fuel consumed by both airlifter and tanker. The key to this study is the requirement that the model be operationally usable. Therefore, the model is designed to allow input of any mission scenario using geographic coordinates.

The various subobjectives presented in this paper are organized with a view towards validating and verifying the model's output. However, some questions are also answered regarding the nature of the aerial refueling optimization problem. An analysis of the experimental design shows that it is extremely difficult if not impossible to develop general "rules of thumb" to minimize total fuel consumed, primarily because of the complex interaction between the control variables within the aerial refueling system. It is found that cargo load, and hence airlifter gross weight, has more effect on total fuel consumed than takeoff fuel. Also, it is found that the objective of minimizing total fuel consumed is not always consistent with that of minimizing the use of tankers.

The model uses the dynamic programming technique during the solution process. In addition, the "Complex" Method of Box is incorporated to optimize, or minimize, stage return functions.

It is anticipated that the use of this model to plan operational aerial refueling missions will result in significant fuel savings to the Air Force.

AN AERIAL REFUELING OPTIMIZATION MODEL
APPLIED TO STRATEGIC AIRLIFT

I Introduction

Background

> A recent SALT negotiator remarked that the Soviets seemed envious of only one American military capability--the routine and seemingly effortless way we refueled, at night and in all weather, our tactical and strategic forces. For some reason, that capability has not received the same appreciation in the Pentagon's E-Ring these past several years [Ref 20:87].

The above quote from General T. R. Milton, USAF (ret.), highlights the importance of our strategic aerial refueling capability and notes that planned substantive improvements to our aerial tanker fleet are not receiving the proper emphasis. As we see tighter fiscal policies evolve in response to the ever-mounting national debt, it is evident that our aerial refueling capabilities will not keep up with the increasing number of aerial refueling-capable aircraft in the Air Force inventory. Two very important improvements to our aerial refueling capability have been either cut from planned levels or delayed--the new KC-10A and the KC-135A re-engining program are both victims of the proposed Fiscal Year 1982 budget (Ref 10:19). The increasing demand for limited aerial refueling resources, plus the continuing problem of jet fuel

availability and cost, demand that these resources be used as efficiently as possible. This study will define and develop a computer model that can be used operationally to optimize the use of our tanker fleet. Although this study specifically addresses the use of tankers by the Military Airlift Command (MAC), other Air Force users, such as the Strategic Air Command (SAC), will find the results equally adaptable to their aerial refueling mission.

Although aerial refueling was first demonstrated feasible in 1929 by the legendary flight of Ira Eaker, Tooey Spaatz, and others, its full impact was not realized until the Vietnam War (Ref 20:87). In Thailand, which saw the daily rendezvous of North Vietnam-bound fighters with an aerial fleet of KC-135s, we began to better understand and develop the full potential of this capability. Aerial refueling had been used for many years in a training role; however, this was the first time the concept had been proven in actual combat. It also showed that aerial refueling had very important applications in other areas besides the traditional strategic bomber support role.

Later, during the Yom Kippur War of 1973, a new important role for aerial refueling began to emerge. The ensuing aerial resupply of Israel by the United States showed that strategic mobility may very well be the key to success during a contingency involving our interests far from home. Operation Nickel Grass, the code name

assigned to the Military Airlift Command's resupply mission, was extremely successful, resulting in the delivery of 22,395 tons of military equipment in 30 days (Ref 1:122). It did serve to point out one very disturbing fact, however, and that was the dependency of any global airlift contingency on the availability of enroute bases for refueling. Due to the political implications of any future conflict, landing rights in foreign countries, even for only refueling, may be denied. Specifically, if Lajes AB, Azores, had not been available for our use during Nickel Grass, another refueling and staging base would not have been available. For this reason, MAC placed its highest priority on developing its already existent (but dormant) aerial refueling capability.

Today, MAC has total strategic airlift aerial refueling (AR) capability. C-5As have been flying operational AR missions since 1975. C-141Bs have been flying training and operational AR missions since the beginning of 1980, and fleetwide modification of C-141As to the AR version will be completed in 1982. The new C-17 will also have AR capability. Future limitations on mobility will not be dependent on having enough AR-capable airlifters, but rather on how the use of our airlift resources will be planned. Competition from other commands for the use of limited tanker support will require these tankers to be efficiently allocated and used.

MAC is presently without a Command policy specifying which missions should be aerial refueled during a contingency (Ref 13). Consequently, contingency planning with analytical techniques is done by using AR only on a random basis. One part of the problem lies in the fact that MAC planners do not have a means of determining the optimal use of tankers. Another, and more basic, problem is that SAC has not made available to MAC planners the number of tankers that will be available to them in the event of a contingency (Ref 13). An optimizing AR planning tool is first needed to allow realistic inclusion of aerial refueling during the planning phase, so that MAC can coordinate with the Strategic Air Command on the exact number of tankers required for a given contingency. This research will provide such a tool.

Present Models and Concepts

The study of incorporating aerial refueling into present operations plans is a continuing concern to HQ MAC/XP, the operations planning function at MAC Headquarters. The Command still lacks definitive guidance, however, on how AR should be optimally used. Three criteria have been used in the past when considering aerial refueling for mission during the planning phase:

1. First, AR is considered if the mission must aerial refuel to avoid enroute stops. This might be to

decrease overall closure time or the fact that a stop might not be possible due to fuel availability or political considerations. In any case, these missions receive first priority for AR during the planning phase.

2. An operational or aircraft limitation will place a mission in the next priority for AR. For example, either runway length or maximum aircraft gross weight limitations may not allow the airlifter to depart with enough fuel to reach its destination.

3. AR is sometimes used solely to decrease the closure time of an operation. This criterion is not used extensively because of the large demand such an operation may have on tankers from other users.

These practices have been reasonably effective in the past; however, an optimization method of selecting missions for AR would be much more useful in terms of minimizing the use of tanker resources and fuel consumption.

MAC also has a simulation model that uses the AR mission scenario to a limited extent. This model is called M-14 and its primary purpose is to test the capacity of the airlift system under varying conditions. It makes use of Monte Carlo simulation techniques to model all facets of MAC airlift, incorporating all strategic airlift aircraft and including such factors as crews, air bases, materiel depots, and logistics materiel used in daily MAC operations (Ref 17). M-14 does include the AR option for

its mission scenarios; however, the missions chosen for AR are done so randomly with no attempt at optimizing the use of tankers. A total of 25 arbitrary AR rendezvous points are built into the model, and the model selects a rendezvous point for a specific mission not to minimize fuel consumption, but to minimize the fuel upload at the cargo onload station. Also, tankers are not modeled in M-14, but instead are treated as unlimited resources. Furthermore, consideration for multiple tanker AR is not allowed (Ref 17). A model which selects optimum rendezvous points and tanker departure bases is needed to enhance M-14's treatment of the AR scenario (Ref 13). It would provide a more realistic and fuel conservative basis within M-14 for using AR, compared to the present arbitrary method of selecting refueling points and fuel offloads.

Research results reported by Captain Bordelon and Major Marcotte (Ref 5) used the Simulation Language for Alternative Modeling (SLAM) package to model the AR mission scenario and were successful in finding an optimum rendezvous point for one specific scenario. The purpose of their research, however, was to define a general AR policy rather than develop a model suitable for operational use. Also, optimization techniques were not used, but rather a total enumeration of feasible rendezvous points was done followed by the selection of the point requiring the least amount of fuel. Certain assumptions were also

made that limited the operational usefulness of the model. For example, multiple tanker AR was not allowed. Also, a typical scenario was built into the model with no provision for inputing actual route segments. Although unsuccessful in defining a general AR policy, the Bordelon-Marcotte study did develop a valid fuel consumption model, portions of which are used in this study.

Problem Statement

Airlift planning for future contingencies must include the aerial refueling option. Furthermore, given limited tanker resources and fuel, aerial refueling mission planning must be done as efficiently as possible. Literally every new aircraft entering the Air Force inventory will be AR-capable; therefore, tanker resources will continue to be limited. Airlift planners presently do not have a method available to them that will provide an optimal solution to the planned rendezvous point and tanker departure base problem. A model that can do this will not only enable airlift planners to better define MAC tanker requirements, but will also result in a substantial overall fuel savings.

Objectives of This Study

The overall objective of this study is to develop a model that will, given a route of flight, enroute wind factors, airlifter takeoff fuel load, available tanker

bases, and bases usable for diversion if AR is unsuccessful, determine the optimal AR rendezvous point and tanker departure base to minimize the total fuel consumed by both airlifter and tanker. To meet the objective, the model must allow the user to select any combination of aircraft (KC-10A, C-5A, C-141B, KC-135A) and must allow multiple tanker aerial refueling. Furthermore, this model must be capable of being used operationally for planning routine AR missions as well as for planning contingency airlift. It must also be adaptable for use with MAC's M-14 model of the airlift system.

A subobjective of this study involves exercising the model in accordance with the experimental design, resulting in an analysis of the sensitivity of total fuel consumed to changes in airlifter takeoff fuel and cargo load. This analysis serves to validate the results of the model while providing the planner with some insight as to what causal factors have the most effect on minimizing total fuel consumed. The following three questions are addressed in the analysis.

1. Considering airlifter takeoff fuel and airlifter cargo load, which factor has the most effect on total fuel consumed? It is hypothesized that cargo load will have the most effect.

2. Is there a direct causal relationship between takeoff fuel and total fuel consumed? MAC's present

policy is to depart the airlifter with the maximum possible fuel load consistent with airfield and airlifter limitations. This practice will be evaluated as to its effects on minimizing total fuel consumption and minimizing tanker support.

3. Using a normal rendezvous and offload method of aerial refueling (as opposed to the "buddy" rendezvous/offload operation in which the tanker accompanies the airlifter over a portion of its flight), does the KC-10A offer a significant fuel savings over the KC-135A? It is hypothesized that the findings will show the KC-135A more efficient (in terms of fuel consumption) during normal rendezvous AR.

It should be recognized at this point that the objective of minimizing total fuel consumed may not always be consistent with minimizing the use of tankers. For this reason, tankers are modeled as resources which are available as needed, but which add a penalty (fuel consumed) for each additional tanker used. The model balances the effect of higher airlifter fuel consumption against additional tanker enroute fuel consumption using dynamic programming techniques. The result is the minimum fuel consumed; however, the number of tankers used in supporting the mission may not be the *minimum*. A second subobjective involves addressing this question.

As a third and final subobjective, further model validation is accomplished by comparing model derived optimal rendezvous points and tanker departure bases with those actually used on operational missions. Model output will be compared with actual computer flight plan data from previous AR missions.

Scope and Limitations

This study is limited to only single-ship airlifter cells; however, multiple tanker cell refuelings will be allowed at each possible rendezvous point. The model also allows only one type of tanker (either KC-10A or KC-135A) for each single-ship airlifter mission. Also, "buddy" AR missions are not included in this model; that is, the tankers must rendezvous with the airlifter, offload their fuel, then return to their respective bases, insuring minimum overhead destination fuel requirements are met. Both C-5A and C-141B aircraft are included in the model.

The model requires the user to have and input the following data:

1. Airlifter route of flight using geographic coordinates (maximum of 20 points).

2. Tanker departure bases to be considered, also using geographic coordinates (maximum of 10 bases).

3. Airlifter diversion bases to be used in case AR is unsuccessful (maximum of 10 bases over the entire route of flight).

4. Wind factor for each leg.

5. Type of airlifter (C-5A or C-141B) and type of tanker (KC-135A or KC-10A).

6. Airlifter cargo load, takeoff fuel load, and required overhead destination fuel. This will require a small amount of preliminary fuel planning on the part of the user.

The model then determines where the airlifter should refuel, the tanker offload, and the tanker departure base to be used in order to minimize total fuel consumed by both airlifter and tanker. It also insures that the airlifter has enough fuel to divert to the closest diversion base if AR is unsuccessful.

Benefits Derived From This Study

The justification for this effort falls into three general areas. First, and most important, it will help MAC planners to define a Command policy on aerial refueling, leading to a realistic definition of tanker requirements. Second, it can later be incorporated into the M-14 computer model to provide a sound basis for which missions are selected for AR. And third, it will provide an interactive computer tool to peacetime operational AR planners

that will determine optimal rendezvous points and tanker departure bases. This will result in more training per flight hour, and should result in considerable fuel savings. Also, since the model can be adapted for use with other aircraft, the results of this study can be used in providing a basis for determining future tanker force requirements.

Overview

Chapter II begins with a brief summary of normal airlift fuel planning requirements, followed by a description of the additional fuel planning required by aerial refueling. The aerial refueling scenario is then conceptualized as a system and the relationship between control and response variables is examined. The assumptions made during model conceptualization are then outlined and justified.

Chapter III deals with the quantification of model parameters and the computerization of the model. It begins with a description of the dynamic programming procedure as applied to the AR scenario, to include the definition of the return and transformation functions. The "Complex" Method of Box optimization algorithm (Ref 6) used in the model is then described, followed by the derivation of fuel consumption models for the C-141B, C-5A, KC-135A, and KC-10A. A discussion of validation and verification completes the model description in Chapter III.

Chapter IV presents the experimental design and results, using a typical airlift scenario for the analysis. No attempt is made to derive general rules to optimize fuel consumption, but rather the results are used to verify and validate the model. Model output is explained and compared with expected results. A sensitivity analysis of certain model parameters on model output is also presented. Chapter V completes the study with conclusions and suggestions for further research.

II System Description

Before describing the aerial refueling scenario as a system, a brief overview of the MAC fuel planning process is required. Detailed fuel planning requirements are outlined in the respective aircraft Technical Orders and MAC regulations (Refs 7, 8, 18, 19). This discussion will be limited to the details required for justifying the present research design. Figure 1 graphically portrays the normal fuel planning sequence for a typical airlift mission. Figure 2 shows the additional fuel requirements for an AR airlift mission. *Figure 3 shows a portion of* an actual computer flight plan depicting the different fuel planning phases. These phases, numbered from one to thirteen, are also shown on Figure 1 for reference.

Normal Fuel Planning

Phases one, two and three of the normal fuel planning sequence deals with the enroute portion of the flight. Enroute fuel is obtained from the applicable fuel planning documents (Ref 19) based on aircraft gross weight, temperature deviation from standard, altitude profile, true airspeed, and flight times. Enroute reserve is added to account for unexpected enroute diversions or inaccurate wind forecasts. This figure is always defined

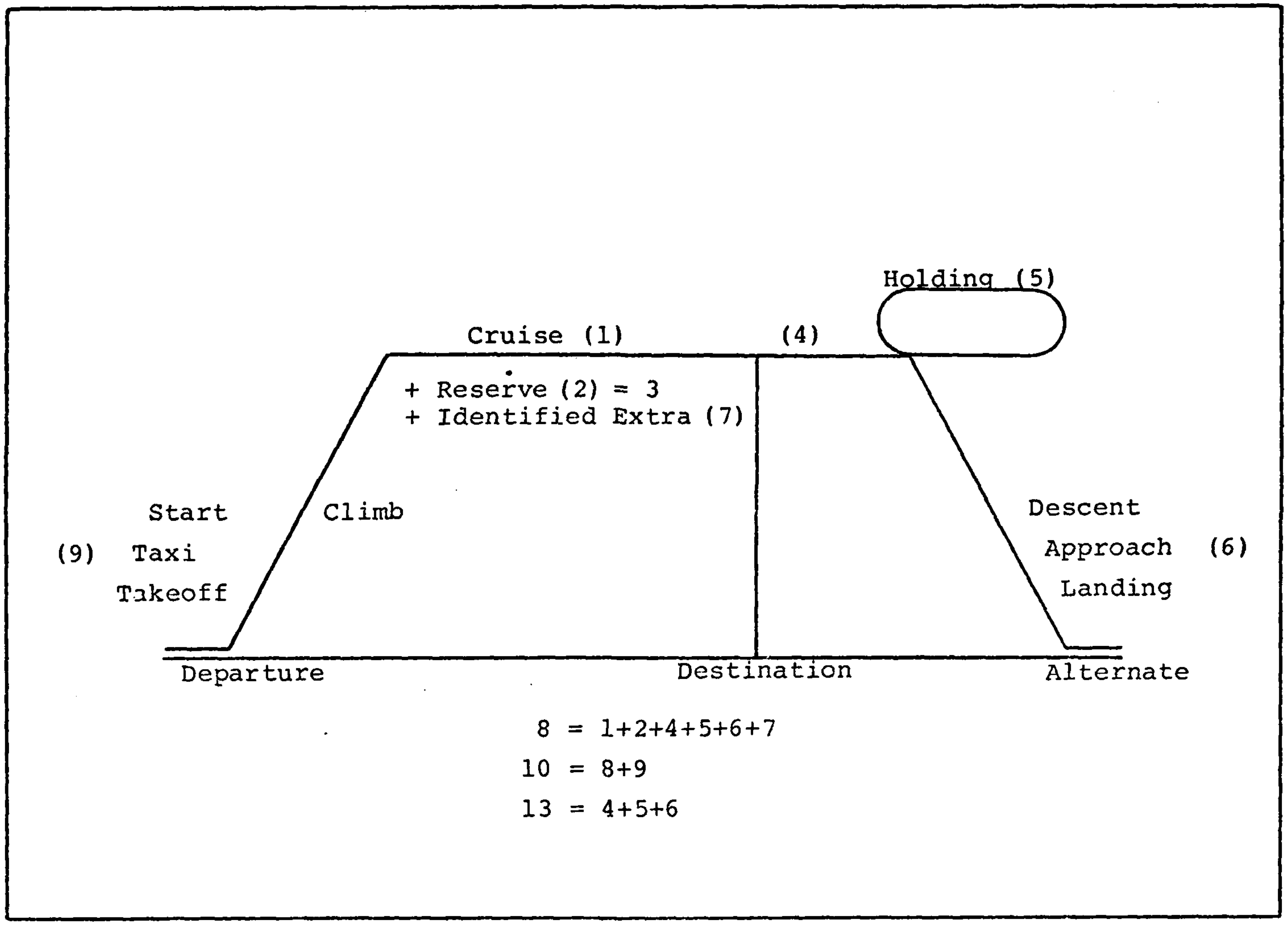


Fig. 1. Normal Fuel Planning

(Ref 18) as 10 percent of the fuel required over the portion of the route where enroute navigation aids are inadequate to accurately position the aircraft at least once each hour (also known as the Category I portion of the route). Enroute fuel also includes the fuel required to climb to initial level-off altitude, and is highly sensitive to aircraft gross weight (which limits altitude capability) and true airspeed.

Alternate fuel, phase four, includes the fuel required to climb away from original destination and cruise to a suitable alternate. The altitude and true airspeed used in planning are determined by the distance from original destination to the alternate. This fuel figure is again obtained from the applicable fuel planning documents (Ref 19) using aircraft gross weight overhead destination, temperature deviation from standard, and air distance to the alternate as inputs. An alternate is always required for overseas airlift missions (Ref 18).

Holding fuel, phase five, is defined as the fuel required to hold for 45 minutes at 10,000 feet at the alternate, if required, or to hold 1 hour and 15 minutes at 20,000 feet at the destination. This fuel figure is based on aircraft gross weight at either destination or alternate and is obtained from aircraft fuel planning documents (Ref 19).

Approach and landing and taxi/run-up fuel, phases six and nine, are assumed constant during the planning phase. For the C-5A, the fuel required for approach and landing is 5200 pounds, and the fuel required for taxi/run-up is 2800 pounds; for the C-141B, the appropriate figures are 2500 and 1900 pounds, respectively.

Phase seven, the identified extra fuel, includes the fuel added for any other reason not covered in the other phases. This would include planned additional taxi time, known enroute diversions or delays due to traffic or weather, or extra fuel carried to avoid refueling at destination. For most missions, this figure is zero.

Phase 13, the required overhead destination fuel, is sometimes referred to as the most important figure in terms of enroute fuel management. The crew must periodically check this planned figure against what the actual overhead destination fuel will be throughout the flight. This figure includes the fuel needed to fly from destination to alternate, hold, and approach and land at the alternate. Therefore, if at any time the crew determines that the actual overhead destination fuel will be less than this planned figure, it must take some action to decrease fuel consumption or consider diversion.

Phases 8 and 10 are totals, and are shown in Figure 1. Phase 11 indicates planned actual fuel load. Phase 12 is no longer included on computer flight plans,

but represents the difference between phases 10 and 11, or unidentified extra fuel.

AR Mission Scenario

The typical AR mission scenario and fuel requirements are outlined in Figure 2. Start, taxi, takeoff, enroute reserve, fuel to alternate, holding, and approach and landing fuel requirements are exactly the same as those for a normal airlift mission. Cruise fuel planning for an AR mission normally requires several entries into the enroute fuel charts of the applicable fuel planning document (Ref 19) for the different altitude and airspeed combinations encountered throughout the mission. The following discussion will illustrate the process involved.

Initial climb fuel from departure point is normally computed to optimum altitude, or the altitude which provides a 300 foot-per-minute rate of climb, unless AR is anticipated immediately after level-off. If this is the case, level-off is accomplished at 25,000-26,000 feet. It has been shown that this altitude range provides the best flight control response for both airlifter and tanker at typical refueling gross weights. The refueling track itself is defined along the airlifter's route of flight, beginning at the initial point (IP) and ending at the end-AR point. The control point (CP) is normally 100 nautical miles from the IP and defines the geographical point the

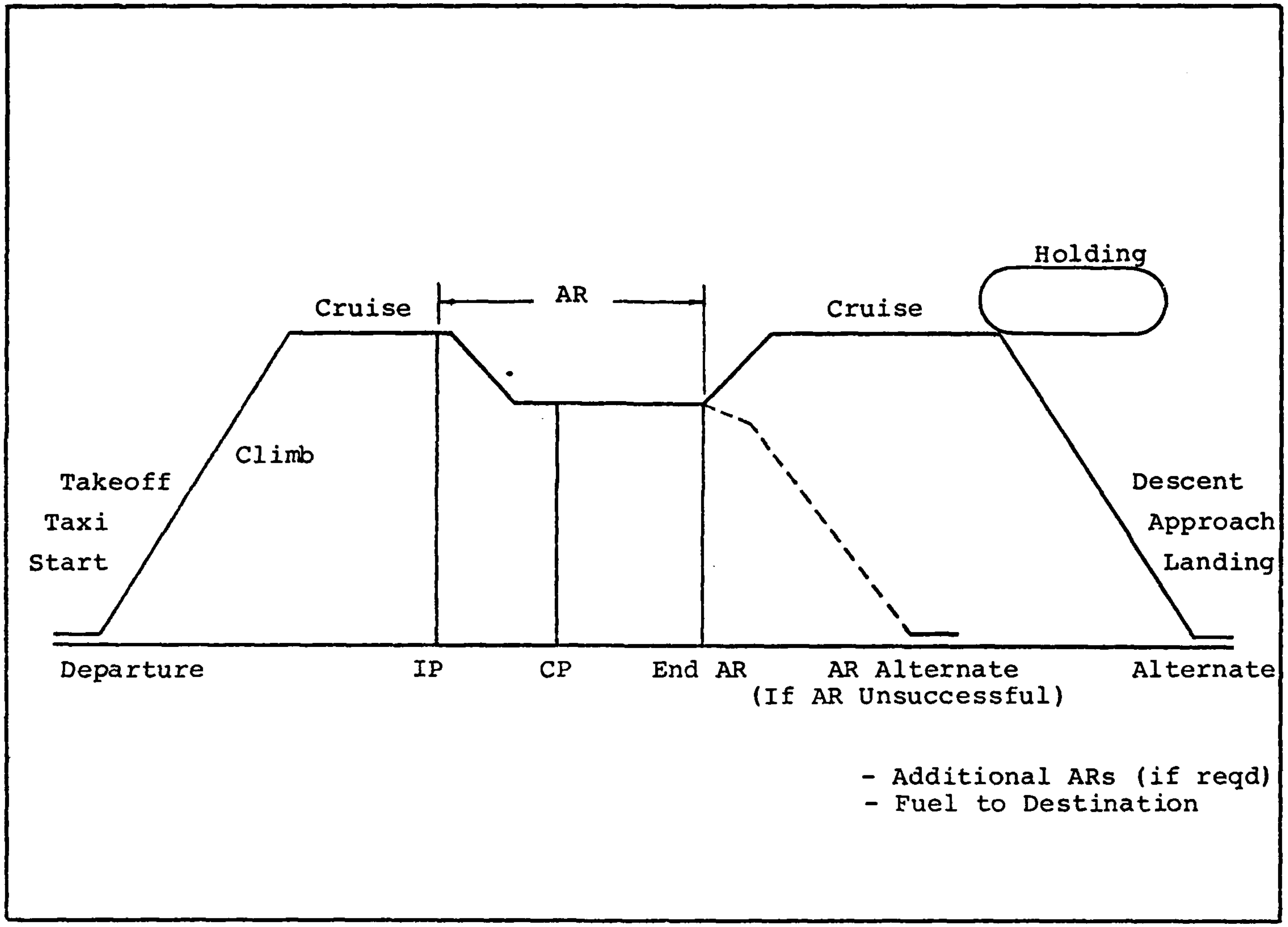


Fig. 2. AR Mission Fuel Planning

AFGWC TIME AND FUEL ANALYSIS (MAC)					
1-ENROUTE	1558	373900	2-RESERVE	0032	012500
3-ENROUTE + RESERVE	1630	386400	4-ALTERNATE	0000	000000
5-HOLDING	0115	023700	6-APP/LANDING	0015	005200
7-ID EXTRA FUEL		000000	8-TOTAL TAKEOFF	1800	415300
9-TAXI/RUNUP FUEL		002800	10-REQUIRED RAMP		418100
11-ACTUAL RAMP		225800	13-OVERDEST FUEL		028900
BURN OFF FUEL		379100	ENDURANCE	0737	

Fig. 3. Computer Flight Plan Fuel Block

tanker and airlifter ideally should have completed their rendezvous. The airlifter crosses the IP at cruise altitude and airspeed and establishes radio contact with the tanker, which should be orbiting at the CP. At 80 nautical miles from the CP, the airlifter descends to 1000 feet below tanker altitude (unless already there), and the tanker departs the orbit and begins a turn towards the airlifter. The aircraft then continue flying toward each other until a pre-established separation is reached (usually 8 nautical miles), at which time the tanker performs a 180 degree turn back to refueling heading, hopefully rolling out directly in front of the airlifter at 2 nautical miles horizontal and 1000 feet vertical separation. Fuel for this entire procedure is determined by entering the fuel planning charts with a planned gross weight, AR altitude, and temperature deviation from standard at altitude. Air refueling commences at the CP at 252 knots indicated airspeed and continues to the end-AR point. The end-AR point is typically 150-400 nautical miles from the CP, depending

on the number of tankers in the refueling cell. After AR is complete, the airlifter climbs again to its optimum altitude, which can now be as much as 10,000 feet lower than its pre-AR optimum. This lower altitude capability, because of the higher airlifter gross weight, becomes a significant factor in locating the optimum refueling points.

One additional fuel requirement for the AR mission scenario requires emphasis. The airlifter must always have enough fuel at the end-AR point to divert to a suitable alternate base if AR is unsuccessful (Ref 18). This requirement defines a lower bound on the fuel state of the airlifter, as described later in Chapter III.

Tanker Fuel Requirements

Tanker fuel requirements are conceptually the same as those for the airlifter. The tanker must have enough fuel to fly to the CP, orbit for one hour (in case the airlifter arrives late at the IP), return to its recovery base, hold for 45 minutes, approach, and land with a specific fuel reserve. Table 1 outlines these requirements as assumed for the model. These figures are assumed constant for each tanker mission, leaving the enroute cruise fuel as the only variable fuel figure. The enroute cruise fuel is a function of gross weight and true airspeed and is determined in the same manner as that for the airlifter.

TABLE 1

TANKER RESERVE REQUIREMENTS

	KC-10A	KC-135A
One Hour Orbit	17322 lb	11263 lb
Climb, Descent, Approach and Landing	13900 lb	7800 lb
45 Minutes Holding	11500 lb	6500 lb
Total	42722 lb	25563 lb

System Conceptualization

The aerial refueling scenario is modeled in Figure 4 as a network, with each node representing a specific geographic coordinate along the airlifter's route of flight. Node 1 then becomes overhead destination, and node N is the point the airlifter assumes its cruise altitude and airspeed profile after departure. The airlifter enters the network with a specified takeoff fuel and must arrive overhead destination with at least planned required overhead destination fuel. At each node, the airlifter can either refuel or choose not to refuel; if the AR option is chosen, the airlifter must then decide how much to offload from the tanker. Each node also has penalties associated with the decisions made, in terms of total fuel consumed. The first is the fuel required for the airlifter to fly the next segment of the route of flight. This is highly

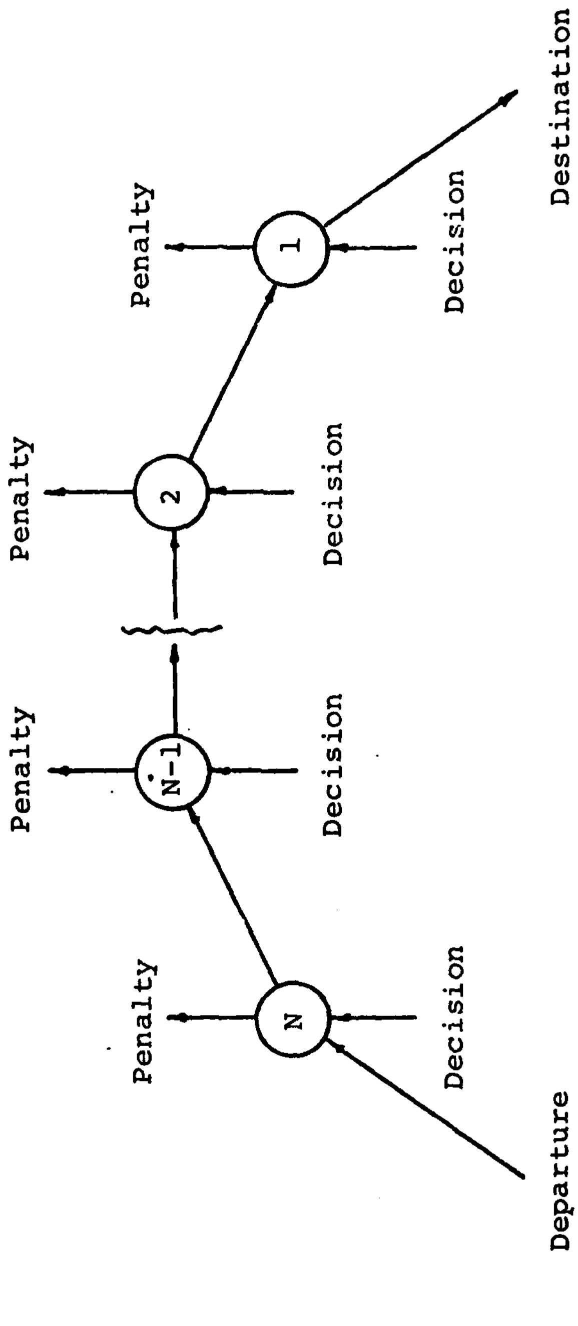


Fig. 4. Network Model

dependent on the refueling decision, because of the strong direct relationship between gross weight and fuel consumption. The second penalty is the fuel consumed by the tanker to fly from its departure base to the node. This again is highly dependent on the refueling decision. The objective of the model is to determine what decisions should be made at each node to minimize the total of these penalties.

The problem is further modeled as a system in Figure 5, showing the control variables and their effect on the response variables. The system boundaries include the airlifter route of flight and all available tanker departure bases specified as input. Tankers are available on demand, but at a significant cost to the system. The measures of effectiveness of the system are the response variables--total fuel consumed and number of tankers used. The control variables, for a given scenario, are the type of tanker used, takeoff fuel weight, cargo load, and refueling decisions possible at each node. The variables fixed for a given scenario are the type of airlifter, airlifter route of flight, wind factor for each leg, airlifter diversion bases, and tanker departure bases.

An analysis of the causal loop diagram shows the refueling decision to be the driving factor for all other control variables affecting the total fuel consumed. The decision to refuel more decreases the takeoff fuel

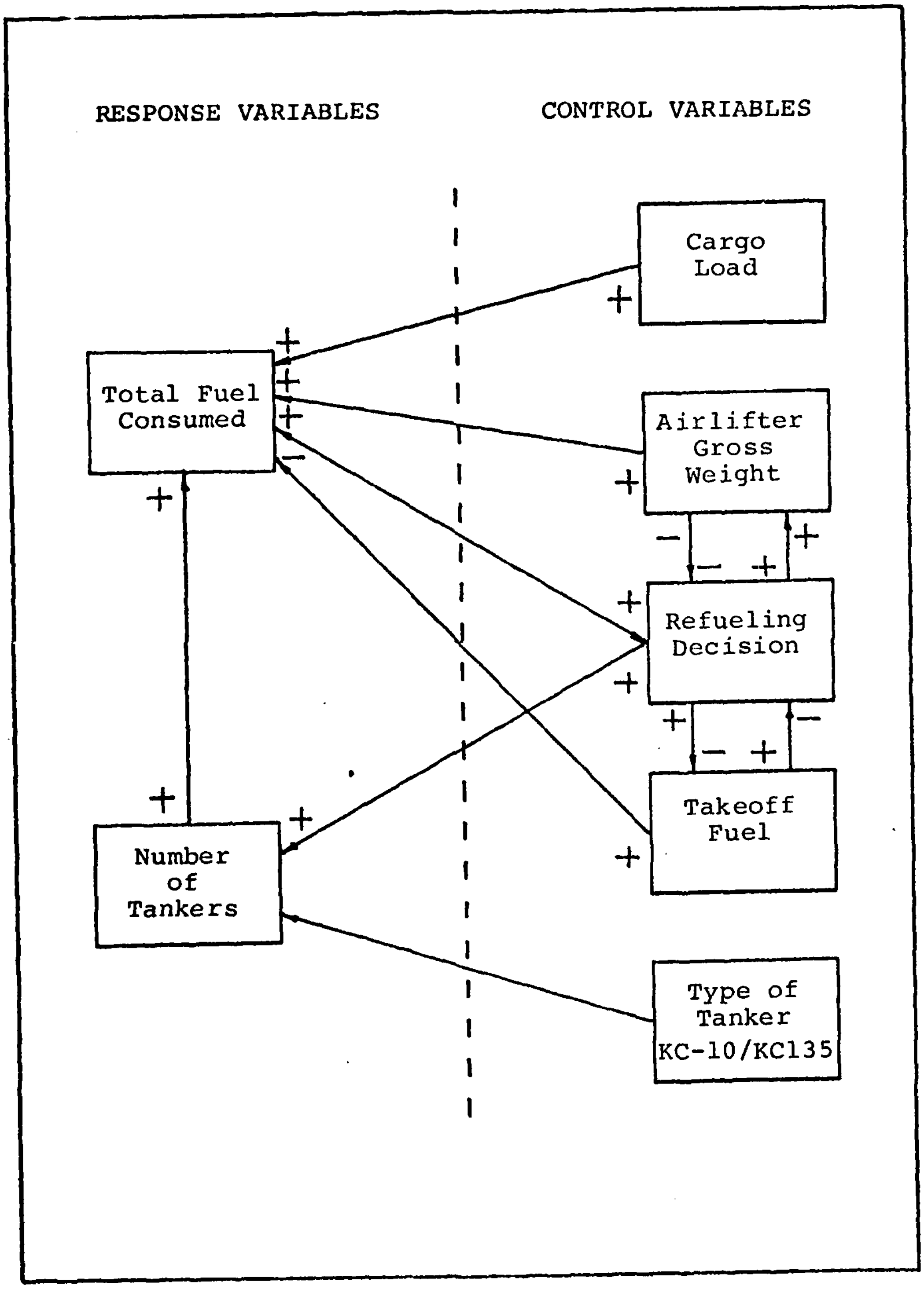


Fig. 5. Variable Relationships

requirement, but in turn increases the gross weight of the airlifter, thereby increasing total fuel consumption. Also, as the AR decisions increase, the number of tankers required increases, also increasing total fuel consumption. It may seem that cargo load has an independent effect; however, further analysis shows that increasing cargo weight increases gross weight, which in turn increases the refueling decisions and again increasing total fuel consumption. This diagram and its interrelationships serve to point out the complexity of the problem to be solved.

Simplifying Assumptions

Because of the complex interactions between the many variables in the problem, certain simplifying assumptions were made and must be justified before describing the details of model formulation. These assumptions are as follows:

1. Tankers will always depart with maximum fuel load. This assumption is consistent with alert aircraft fuel loads and the objective of minimizing tanker use. The tanker can offload any amount of fuel to the airlifter, as long as it retains enough fuel to satisfy reserve requirements as specified earlier. These reserve requirements are assumed constant for each tanker mission.

2. The tanker and airlifter fuel for climb, descent, required overhead destination, and approach and

landing is assumed constant during the optimization. The only variable to be optimized, then, is enroute cruise fuel. This is consistent with actual mission fuel planning practices.

3. Tankers will fly great circle direct routings from their departure bases to the AR control point and will recover at the same base in the same manner. The airlifter also will fly great circle routings between each point on its route of flight.

4. The possibility of AR is only allowed at each geographical point defining the airlifter's route of flight. Depending on the length of the mission, this allows a decision to be made every 150-600 nautical miles. Since the typical aerial refueling track is 250-500 miles long, this assumption does not have a significant effect on the optimal solution. It does, however, considerably decrease the computational time required.

5. Fuel used by the airlifter during the aerial refueling procedure is assumed to be the same as if the airlifter were at enroute cruise. The offsetting effects of lower true airspeed during the refueling maneuver (resulting in lower fuel consumption) and lower altitude (resulting in higher fuel consumption) make this assumption reasonable.

6. Wind factors are assumed to remain constant over each leg of the airlifter's route of flight. This

assumption is again consistent with normal fuel planning practices.

These assumptions, together with the limitations presented in Chapter I and the system description contained in this chapter, fully define the problem to be solved. In summary, the problem can now be defined as a network, with nodes representing points along the airlifter's route of flight, and the output of the system equal to or greater than the required overhead destination fuel. Each node has a decision associated with it and a cost associated with that decision. The objective is to minimize the total cost of the system.

III The Model

Dynamic Programming Concepts

Dynamic programming is a mathematical technique often useful for making a sequence of interrelated decisions. It provides a systematic procedure for determining the combination of decisions that maximizes overall effectiveness (Ref 14:266). Richard Bellman further describes the problem ideally suited to the dynamic programming solution technique (Ref 3:81):

1. In each case, there is a physical system characterized at every stage by a small set of parameters, the state variables.

2. At each stage, there is a choice of a number of decisions.

3. The effect of a decision is a transformation of the state variables.

4. The past history of the system is of no importance in determining future actions.

5. The purpose of the process is to maximize (minimize) some function of the state variables.

Bellman's fourth point is the most significant when applied to the AR optimization problem. It says that whatever the initial decisions are, the remaining decisions

will result in an optimum with regard to the first decision. The idea of sequencing decisions enables a complex problem of many decisions to be broken down into a sequence of smaller problems with only a few variables (Ref 16:183). Each of these smaller problems is called a state, with its optimal decisions resulting in a return to the system's total objective function. Each stage also has a state value associated with it, representing the state of the system at that stage. The stage state values are linked together by a transition function which defines how the system state changes from stage to stage. Therefore, the effect of a policy decision at each stage is to transform the current state into a state associated with the next stage, using the transition function.

Figure 6 shows the typical dynamic programming problem in a serial stage format; that is, each stage occurs in direct sequence. S_n represents the system input state, with S_i representing the state value at stage i. The variable d_i is the stage decision, and r_i is the return at stage i given decision d_i. $\tilde{S}_i$ is the state transformation function, and S_o the system output state. The problem becomes one of finding the optimum of a function of the stage returns subject to constraints on the decision variable:

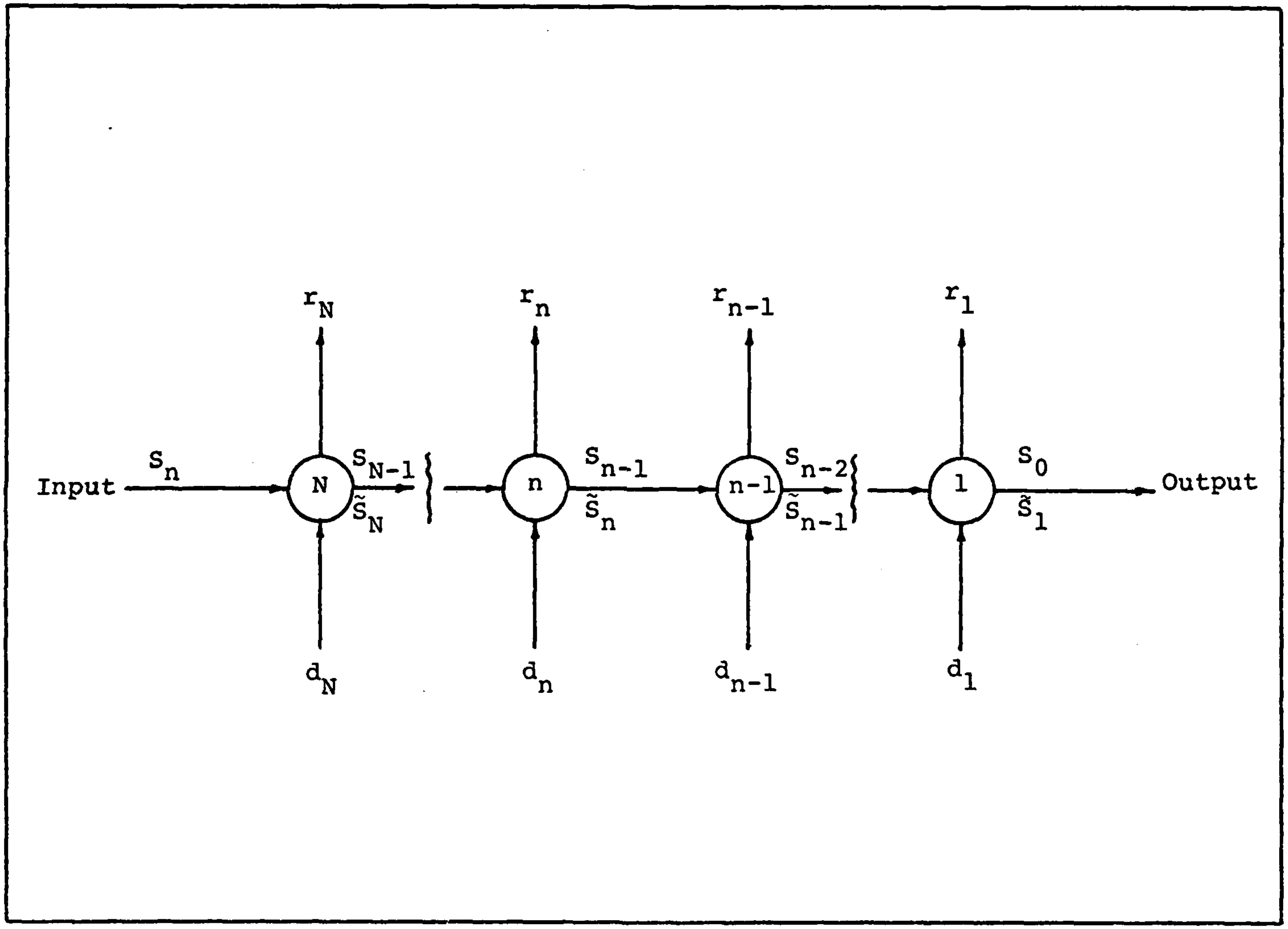


Fig. 6. Problem Structure

$$\text{Minimize} \quad F(r_1, r_2, \ldots, r_n)$$

$$\text{Subject To} \quad G_j \leq d_j(S_j) \leq H_j \quad j=1,2,\ldots,n \qquad (1)$$

where n is the number of stages, G_j is the lower limit of d_j, the decision variable, and H_j is the upper limit. The state variables are in turn constrained, either between constants or functions of the input and output states.

Formulation of the AR Problem

The AR optimization problem is ideally suited to the use of dynamic programming solution techniques. To illustrate, consider a problem with 20 possible AR rendezvous points, each point representing a decision to refuel or not to refuel. Using network analysis optimization techniques, a total of 2^{20}, or 1,048,576 possibilities would have to be considered (Ref 2) (see Figure 7). Since dynamic programming effectively eliminates non-optimal solutions at each stage (recall Bellman's fourth point discussed in the previous section), only a small subset of the total number of possibilities must be considered at each stage. This "principle of optimality" allows the solution of a much larger problem with considerably less computer memory and time.

Figure 6 can again be used to illustrate dynamic programming formulation of the AR optimization problem. Each stage represents a point along the airlifter's route of flight, and the state variable at each stage is the

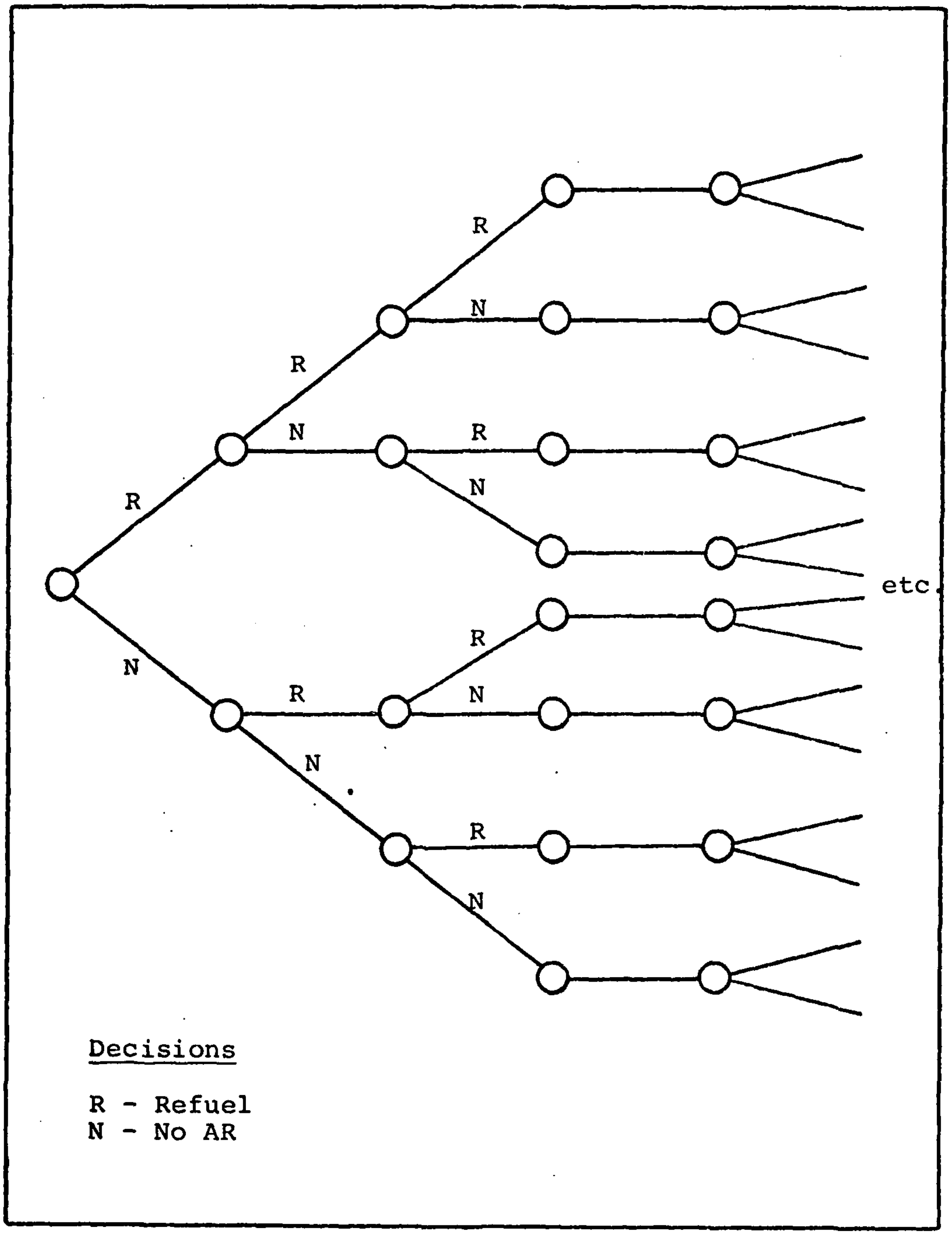


Fig. 7. Network Solution

airlifter's fuel state prior to making the refuel/no refuel decision. Therefore, each stage is also a possible AR rendezvous point. The decision variable represents how many tankers are needed for refueling at each stage based on the tanker's available fuel for offload. For example, a decision value of 1.26 would require 2 tankers--one tanker would offload all its available fuel and the other only .26 of its available fuel. A decision value of zero (the model actually returns a value of $1x10^{-6}$) would indicate that AR would not take place at that point. The fuel available for offload for each tanker is computed by subtracting the fuel required to fly from departure point out to the AR point and back plus required tanker reserves (see Table 1), from the tanker's maximum fuel weight.

The return function for each stage is the cost associated with the decision made at each stage. To be consistent with the objective of minimizing total fuel consumed, this becomes the fuel consumed by the tanker plus the fuel used by the airlifter to fly the next segment to the next stage. The transition function transforms the current state to the next state by subtracting the fuel used by the airlifter between the stages and adding the fuel onload from the tanker.

At each stage, the airlifter must have enough fuel to at least fly the next route segment plus divert from the next stage to a suitable alternate if AR is unsuccessful

at that stage. This defines the lower bound on the decision value. The upper constraint is driven by the maximum allowable gross weight or maximum fuel load of the airlifter. The state variables are constrained by the same factors. Further details of how the constraints are entered in the model are located in Appendix B.

Quantification

Aircraft Parameters. Table 2 lists the appropriate aircraft limitations and parameters used in the model. These figures come directly from the respective aircraft performance manuals.

TABLE 2

AIRCRAFT PARAMETERS

Parameter	C-5A	C-141B	KC-135	KC-10A
Max Gross Weight (lb)	712500	323000	270000	590000
Max Fuel Weight (lb)	318100	153352	163000	345400
Cruise True Airspeed (kt)	450	432	479	479
Operating Weight (lb)	354000	148800	105000	248470

Fuel Consumption Models. Gosch and Mooz (Ref 12: 20), in their study of an Air Force Fuel Consumption Model, develop an empirical method of calculating aircraft fuel consumption in gallons per hour. By using multiple regression techniques, they were able to derive a simple relation

that would calculate fuel flow as a function of aircraft gross weight and true airspeed. The general form of the model used in their analysis is

$$FF = A(GW)^{B}(TAS)^{C} \tag{2}$$

where A, B, and C are constants, GW is the aircraft gross weight in pounds, TAS is the true airspeed in knots, and FF is the fuel flow in gallons per hour. Aircraft altitude, a significant factor in determining fuel consumption, is accounted for by the gross weight term, since a gross weight defines a specific altitude capability. This general model was used to derive specific models for the C-5A and C-141B, KC-135A, and KC-10A, using the Statistical Package for the Social Sciences (SPSS) regression routines (Ref 4). The resulting models for each aircraft are:

$$\text{KC-10A:} \quad FF = 7.995\text{x}10^{-6}(GW)^{.9795}(TAS)^{1.1214} \tag{3}$$

$$\text{KC-135A:} \quad FF = 2.54\text{x}10^{-8}(GW)^{.916}(TAS)^{2.24} \tag{4}$$

$$\text{C-5A and C-141B:} \quad FF = 1.81\text{x}10^{-3}(GW)^{.8345}(TAS)^{.5503} \tag{5}$$

A factor of 6.5 pounds per gallon is used to convert the fuel flow in each case to the more commonly used pounds per hour. A full description of the regression analysis is presented in Appendix A.

The fuel actually consumed over a route segment is calculated using the following simple relation:

$$\text{Fuel Consumed} = [XRT/(TAS+WF)] * FF, \tag{6}$$

where FF is the fuel flow as defined above, TAS is the true airspeed in knots, XRT is the length of the route segment in nautical miles, and WF is the wind factor over that route segment. Appendix B describes the model used to calculate the great circle distance between two points from the geographic coordinates.

Return Function. Using the analysis in the previous section, the system return for each stage is the fuel required by the tanker plus the fuel required by the airlifter to fly the route segment from that stage, or

$$r_i = \langle d_j \rangle * (XMAXFT-FTNK_i) + [(FF_i*XRT_i)/(TAS+WF_i)] * 6.5 \tag{7}$$

where

d_i = decision at stage i

$\langle d_i \rangle$ = integer number of tankers required to deliver the offload. $\langle d_j \rangle$ is the integer greater than or equal to d_j

XMAXFT = maximum fuel weight (takeoff fuel) of the tanker

$FTNK_i$ = the fuel available for offload at stage i

r_i = the return for stage i

The fuel flow is calculated using the airlifter's gross weight at each stage after refueling.

Transition Function. The state transformation between stages is determined from the following:

$$SMAX_i = SMAX_{i+1} - [(FF_i * XRT_i)/(TAS+WF_i)] * 6.5 + [d_{i+1} * FTNK_{i+1}] \quad (8)$$

where $SMAX_i$ is the airlifter fuel state at stage i and $SMAX_{i+1}$ is the airlifter fuel state at the previous stage. The fuel flow is again calculated using the airlifter's gross weight at each stage after refueling.

System Return Function. The system return function used in the optimization procedure is simply the sum of the individual stage returns as defined in eq (7), or

$$F = \sum_{i=1}^{n} r_i \quad (9)$$

for the n stage problem.

Tanker Fuel Available for Offload. Fuel available for offload to the airlifter is calculated at each stage from the following expression:

$$FTNK_i = XMAXFT - \{[(FF * XTNK_i)/TAST] * 2 + RES\} \quad (10)$$

where

XMAXFT = the maximum allowable fuel load for the tanker

FF = tanker fuel flow in pounds per hour

$XTNK_i$ = great circle distance in nautical miles from the closest tanker departure base to stage i

TAST = tanker true airspeed in knots

RES = required tanker reserves

The gross weight used in the fuel flow calculation is the tanker operating weight plus one-half of the maximum fuel load. This results in an average fuel flow and assumes that the tanker burns or offloads all fuel during its mission.

Solution Algorithm

The dynamic programming algorithm proceeds as follows (refer to Figure 6):

1. At stage 1, the upper and lower limits on the input fuel state are evaluated. The interval from the high limit to the low limit is divided into a finite series of equidistant steps.

2. Beginning at the lower limit on the state, S_1, the optimum return is found over the range of decisions d_1:

$$f_1(S_1) = \underset{d_1}{\text{opt}} \{r_1(S_1,d_1)\} \tag{11}$$

The "Complex" Method of Box (see next section and Appendix D) is used to find the optimum return for each stage.

3. The state value is then incremented and the optimum is found for this value of S_1.

4. The process is repeated until the upper bound on S_1 is reached.

5. Starting with n=2, the process moves backwards through the stage sequence. The limits on S_n are evaluated and the state interval is divided into equidistant steps.

6. The optimum return is found for each value of S_n by the formula

$$f_n(S) = \underset{d_n}{\text{opt}} \{r_n(S_n,d_n) + f_{n-1}(S_n,d_n)\}. \qquad (12)$$

7. Recursion is followed until stage n is reached and the optimum found for all values of S_n.

8. The algorithm then follows the optimum path forward to recover the optimum decisions as follows: The input fuel state S_n is located in the table of state values for stage n, with the corresponding f_n and optimum decision. The transition function between S_n and S_{n-1} is used to find the optimum S_{n-1} and its corresponding values. This forward path is followed through all stages until S_1 and the corresponding r_1 and optimum decisions are found (Ref 16:185).

<u>The "Complex" Method of Box</u>

The "Complex" Method of M. J. Box (Ref 6) finds the optimum value of a multivariable, nonlinear function subject to nonlinear inequality constraints:

$$\begin{array}{ll} \text{Minimize} & F(X_1,X_2,\ldots,X_n) \\ \text{Subject To} & G_k \le X_k \le H_k \quad k=1,2,\ldots,m \end{array} \tag{13}$$

The implicit variables $X_{n+1},\ldots,X_m$ are dependent functions of the explicit independent variables $X_1,X_2,\ldots,X_n$. The upper and lower limits H_k and G_k are either constants or functions of the independent variables. The method itself is a sequential search technique which has been proven successful in solving nonlinear objective functions with nonlinear inequality constraints. The procedure should tend to find the global maximum by selecting an initial set of points that are randomly scattered through the feasible region. The algorithm evaluates the objective function and searches through the feasible region for an optimum value, starting with a random initial set of points, until it satisfies user defined convergence criteria. A detailed description, along with selected solution parameters, is presented in Appendix D.

Computerization

Appendix C contains a complete FORTRAN V computer listing of the AR optimization model, to include explanations of all variables and subroutines. Sample raw and formated output is presented in Chapter IV and Appendix E. User input instructions are included as comments in the main DYNAM program.

Verification and Validation

Fishman and Kiviat (Ref 11) divide the process of model evaluation into three categories:

1. Verification, to insure that the model behaves as intended;

2. Validation, to test agreement between the behavior of the model and the system it is meant to portray; and

3. Problem analysis, which analyzes and interprets the data generated by the experimental design.

In other words, the primary concern is with the internal consistency of the model, its correspondence with the real system, and correct interpretation of the resulting data (Ref 21:210). History has seen three different viewpoints emerge on the correct method to conduct scientific inquiry--the rationalist, the empiricist, and the absolute pragmatist points of view (Ref 21:212).

The rationalist can be seen on one end of the philosophical spectrum of model evaluation. According to the rationalist, the validity of the model is dependent solely on whether one accepts the basic premises and logic used to construct the model. No proof or experimental measurement of the premises is required. Because of this lack of proof, the rationalist viewpoint is unacceptable to the empiricist at the other end of the spectrum. Empiricism refuses to allow any premises or assumptions that are not

verified by experiment or analysis of experimental data. This way of thinking is extremely limited in application, since it requires a model to begin with proven or verifiable facts, and assumptions are not allowed (Ref 21:210). The third theory of model evaluation, absolute pragmatism, looks at the model as a black box and is only concerned with the usefulness of the output of the black box, not its internal structure. The model, then, is valid only if it fulfills the purpose for which it was designed. Again, this viewpoint overlooks the very important conceptualization phase of model formulation. What is needed is an approach that combines the merits of all three of these philosophies, a sort of verification/validation systems approach, allowing these viewpoints to merge and interact. Shannon (Ref 21:215) calls this the utilitarian approach to the model verification process. This approach is used to verify and validate the AR optimization model.

The first stage of the utilitarian approach is to seek face validity of the internal structure of the model based on a priori knowledge, past research, and existing theory (Ref 21:215). The internal structure, interaction between variables, and analysis of the AR scenario as a system have been examined in the conceptualization phase with a view towards showing face validity of the model. Assumptions have been stated and justified. The

application of dynamic programming and the "Complex" Method of Box will complete this stage in Chapter IV.

The second stage is also concerned with the validation of the internal structure of the model, and consists of empirically testing the assumptions and hypotheses used (Ref 21:216). This stage will also be satisfied in the next chapter when the model is exercised in accordance with the experimental design and model output is compared with expected results. Also, certain model solution parameters will be varied to test the model's sensitivity to these parameters.

The third stage attempts to verify the model's ability to predict the behavior of the real world system (Ref 21:216). This will be accomplished by comparing model derived optimal rendezvous points and fuel consumption with those actually used on operational missions. The philosophical question of how important it is that the model be a true and isomorphic reflection of the real system will also be addressed.

IV Experimental Design and Analysis

The purpose of this chapter is to outline the design of the experiment used to validate the AR optimization model and to present the results of the analysis. This will be done in four parts. First, the experiment used to analyze the effects of the control variables on total fuel consumed will be described, followed by a discussion of the three questions posed in the first subobjective. Then, the question of whether minimizing total fuel consumed is consistent with minimizing the use of tankers will be addressed. A sensitivity analysis of optimization parameters will then be presented, followed by a comparison of model output with actual operational mission computer flight plans. The combination of these four analyses will complete the model verification and validation as outlined in Chapter III.

Part One

Experimental Design. A typical aerial refueling scenario was chosen for analysis, although the model accepts any possible combination of airlifter routings and tanker departure bases. Although the selection of one specific routing and set of tanker bases limits the results of this analysis in terms of deriving a general

optimal rule for selecting AR rendezvous points, it does serve to validate the model and its assumptions. It is felt that because of the limitless number of combinations of routings and possible tanker departure bases, a meaningful general rule would not apply in all cases.

The scenario chosen is shown in Table 3 and involves an airlift mission from Biggs Army Airfield, El Paso, Texas, nonstop to Dhahran, Saudi Arabia. The routing was taken from an actual computer flight plan and reflects International Civil Aviation Organization (ICAO) preferred routings through Europe and the Middle East. Table 3 also shows the assumed wind factors (also obtained from the computer flight plan) and the model generated great circle distances between each point. The coordinates are given in degrees-tenths of degrees for the latitude and longitude, where a negative coordinate represents east longitude and south latitude. Table 4 shows the tanker departure bases and airlifter diversion bases used in the experiment. Figure 8 shows the route of flight, tanker departure bases, and diversion bases geographically.

Table 5 summarizes the experimental design factors and levels for the first phase of analysis. A maximum takeoff gross weight of 650,000 pounds is assumed for the C-5A because of airfield limitations; for the C-141B, the maximum takeoff gross weight is assumed to be 318,800 pounds. Cargo weights for both aircraft are typical

TABLE 3

EXPERIMENTAL DESIGN DATA

AIRLIFTER ROUTE OF FLIGHT:

STAGE	COORDINATES		WIND FACTOR	DISTANCE (NM)
1	[illegible]	[illegible]	22.	[illegible]
2	[illegible]	-35.42	37.	[illegible]
3	32.53	-25.25	13.	[illegible]
4	[illegible]	-12.57	22.	[illegible]
5	42.44	1.42	21.	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	27.	355.52
8	[illegible]	[illegible]	27.	[illegible]
9	52.58	66.51	29.	[illegible]
10	48.15	89.25	11.	[illegible]
11	[illegible]	[illegible]	29.	[illegible]
12	[illegible]	[illegible]	[illegible]	[illegible]

TABLE 4

EXPERIMENTAL DESIGN DATA

TANKER DEPARTURE BASES:

BASE NO.	COORDINATES		BASE
1	34.63	99.33	Altus
2	46.33	87.43	K. I. Sawyer
3	47.43	66.35	Loring
4	4[illegible].[illegible]	3.58	Torrejon
5	52.23	-.73	Mildenhall

AIRLIFTER DIVERSION BASES

BASE NO..	COORDINATES		BASE
1	31.78	1[illegible]6.45	Biggs AAF
2	34.63	99.73	Altus
3	3[illegible].16	71.5[illegible]	Dover
4	53.31	6[illegible].[illegible]5	Goose
5	64.13	21.65	Keflavik
6	4[illegible].46	3.58	Torrejon
7	26.36	-48.[illegible]	Dhahran
8	38.[illegible]	-23.63	Athens

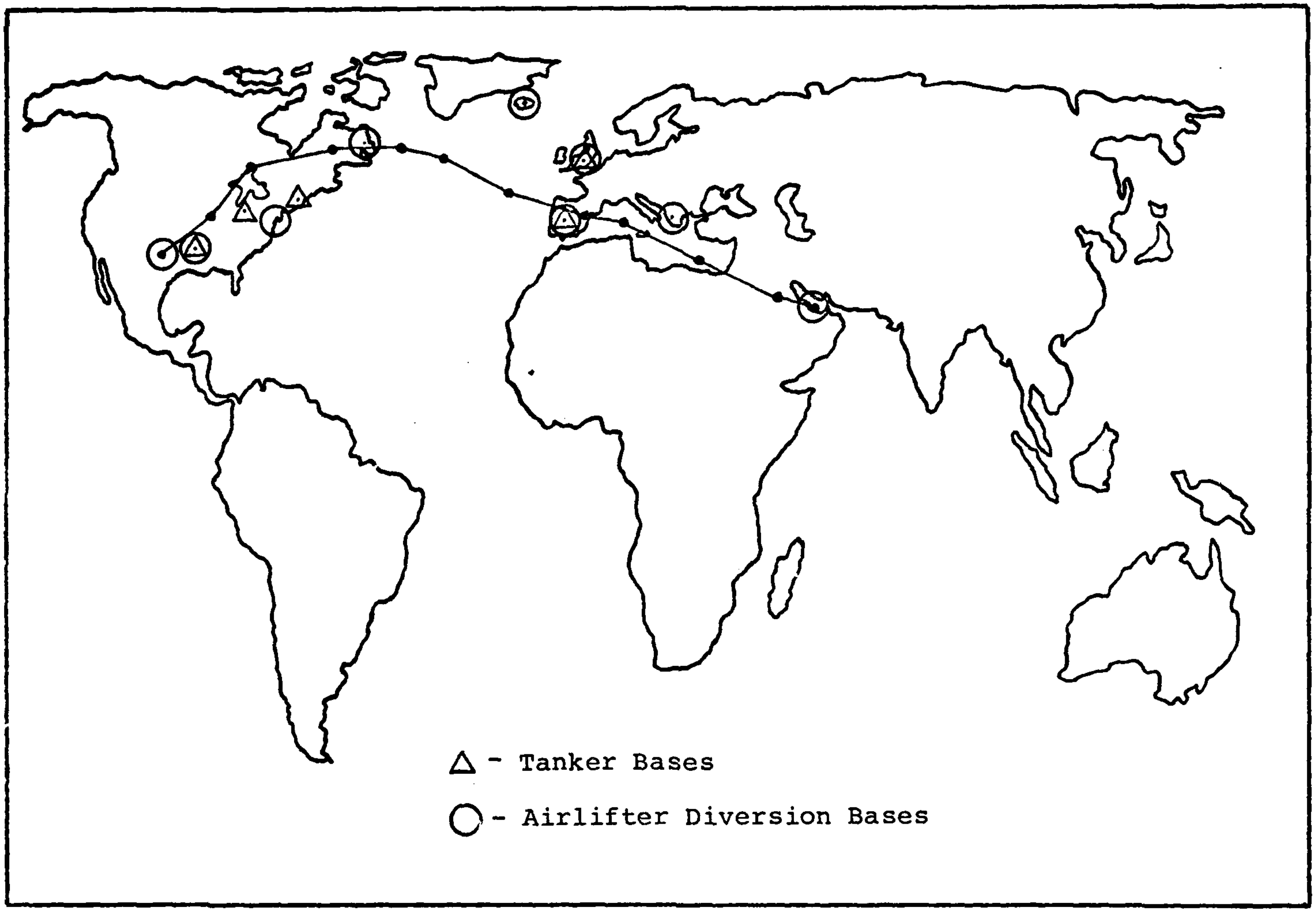


Fig. 8. Airlifter Route of Flight

TABLE 5

EXPERIMENTAL DESIGN LEVELS

Factor	Level
Aircraft Combination	a) C-5A/KC-135A
	b) C-5A/KC-10A
	c) C-141B/KC-135A
	C-5A:
Airlifter Cargo Load	a) 100,000 lb
	b) 125,000 lb
	c) 150,000 lb
	C-141B:
	a) 20,000 lb
	b) 35,000 lb
	c) 50,000 lb
	C-5A:
Airlifter Takeoff Fuel	a) 46,000 lb
	b) 71,000 lb
	c) 96,000 lb
	d) 121,000 lb
	e) 146,000 lb
	f) 171,000 lb
	g) 196,000 lb
	C-141B:
	a) 30,000 lb
	b) 45,000 lb
	c) 60,000 lb
	d) 75,000 lb
	e) 90,000 lb
	f) 105,000 lb
	g) 120,000 lb

productive loads for a mission of this type. Takeoff fuel figures start at 46,000 pounds for the C-5A and 30,000 pounds for the C-141B, and are incremented 25,000 pounds and 15,000 pounds, respectfully, up to a maximum consistent with the selected takeoff gross weights. The C-5A is allowed to refuel from either type tanker; however, the C-141B only refuels from the KC-135A. The C-141B/KC-10A combination is considered infeasible for the rendezvous/offload method of AR because of the tanker's large refueling capability and fuel consumption, and the airlifter's limited ability to accept large amounts of fuel at one time. Since the model is deterministic, only one run of each of the 63 data points was made.

Analysis. Appendix G contains a listing of all model output for the experimental design. Figures 9, 10, and 11 contain plots of total fuel consumed and fuel offloaded for the C-5A/KC-135A and C-5A/KC-10A combinations, all as a function of takeoff fuel and cargo load. Figures 12, 13, and 14 contain the same plots for the C-141B/KC-135A combination.

For all airlifter/tanker combinations, Figures 9 through 14 show that cargo load appears to have the greatest effect on total fuel consumed. This conclusion was tested statistically for the C-5A data, the results of which are contained in Appendix G. As was expected, cargo

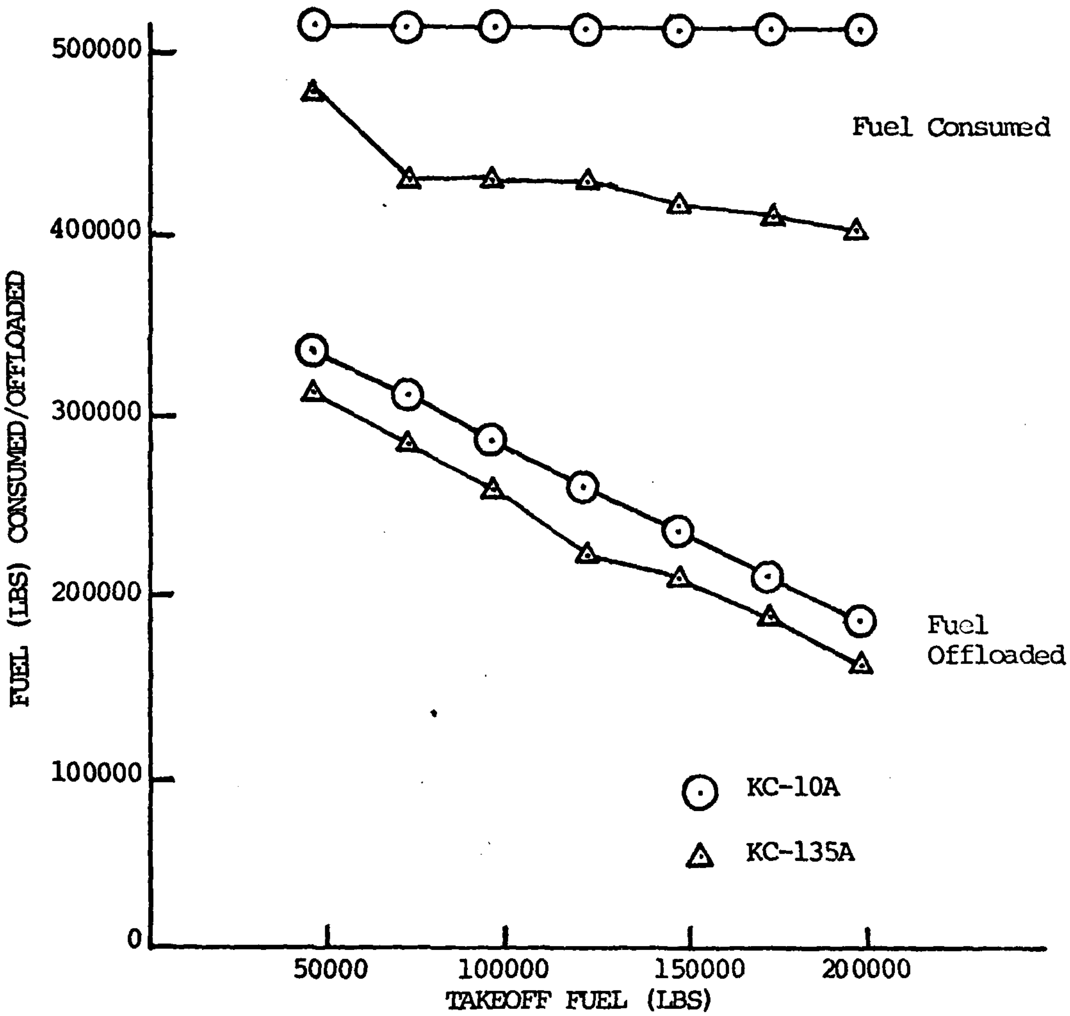


Fig. 9. C-5 Fuel Consumed/Offloaded
Cargo Load = 100,000 lbs

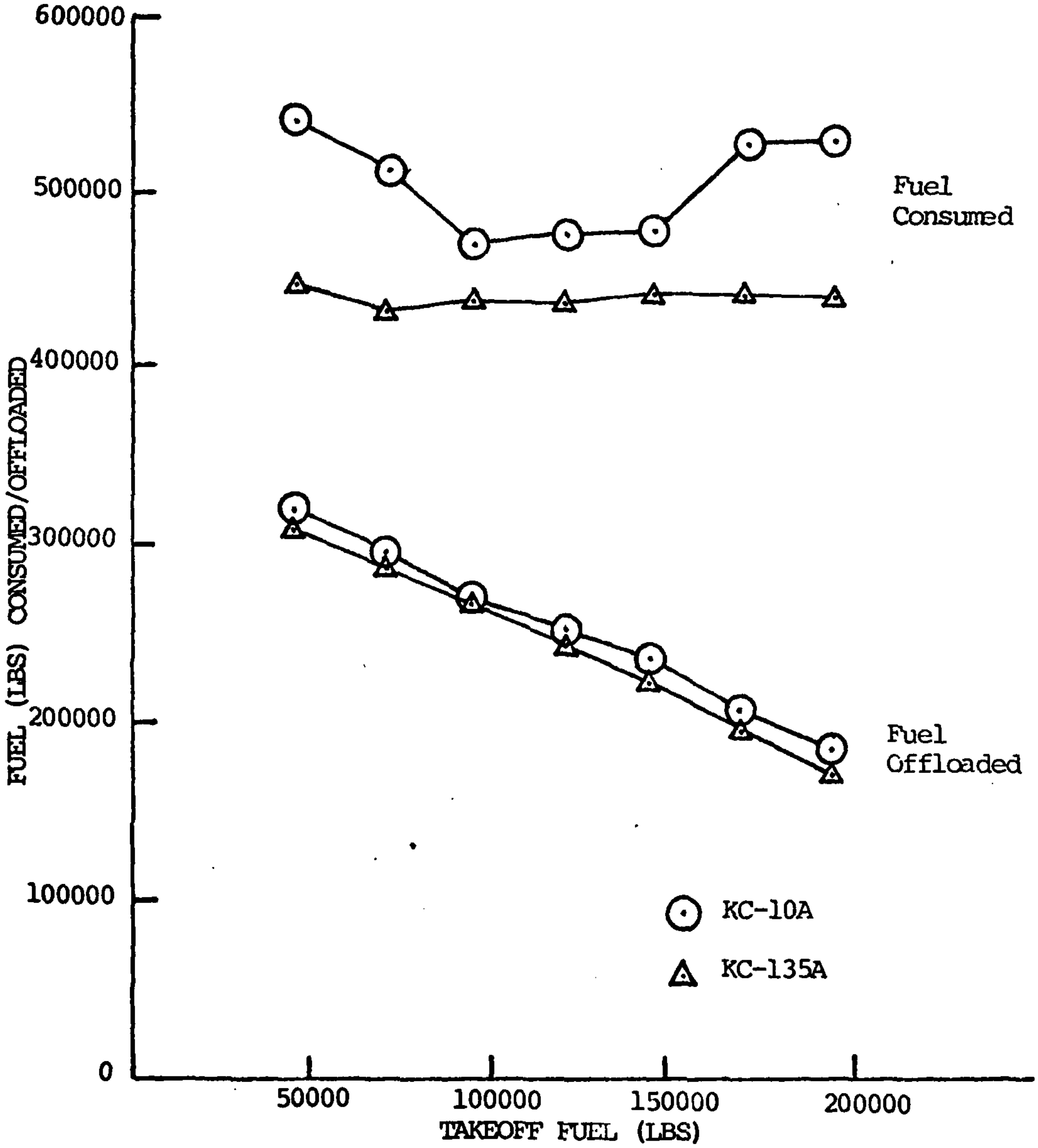


Fig. 10. C-5 Fuel Consumed/Offloaded
Cargo Load = 125,000 lbs

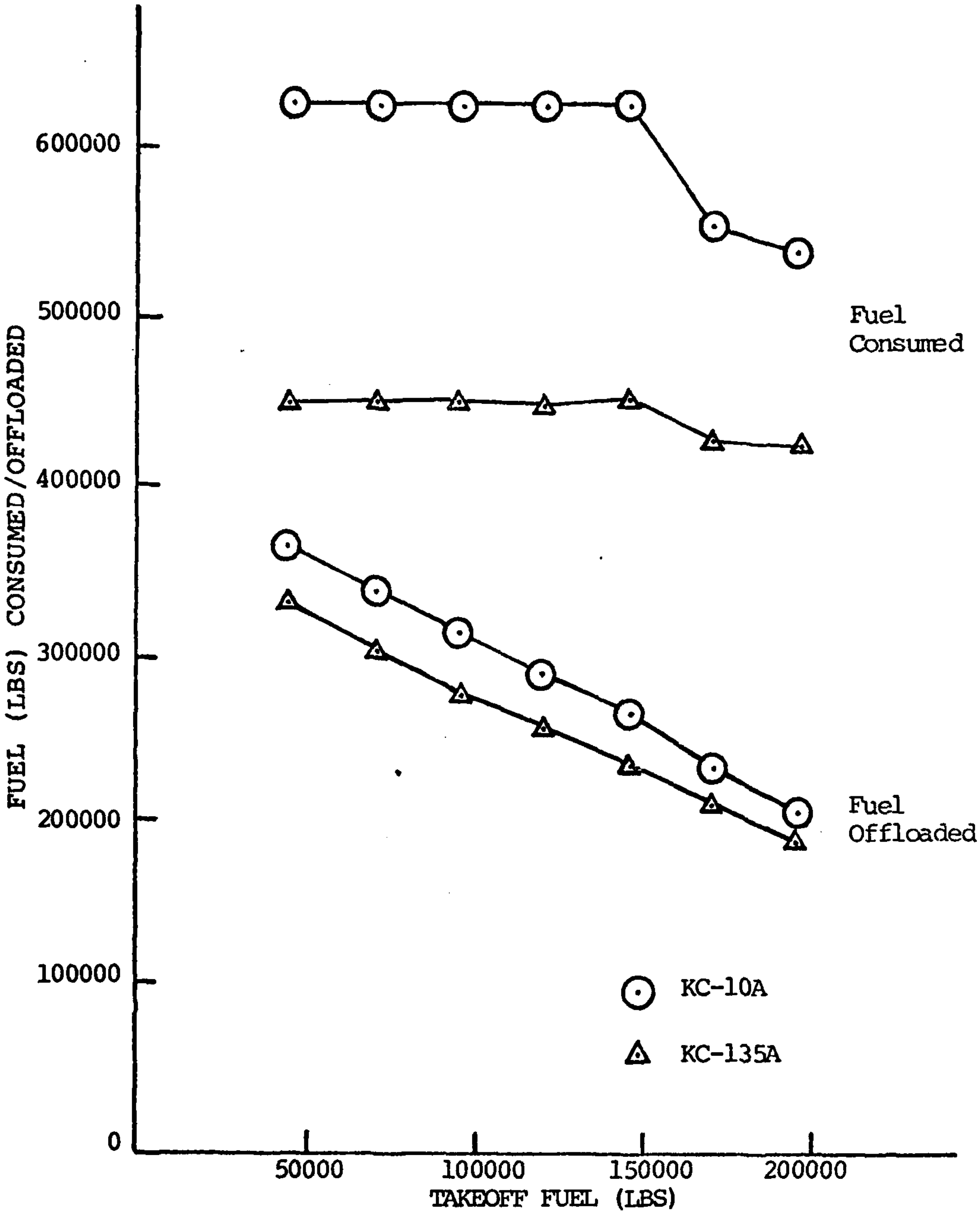


Fig. 11. C-5 Fuel Consumed/Offloaded
Cargo Load = 150,000 lbs

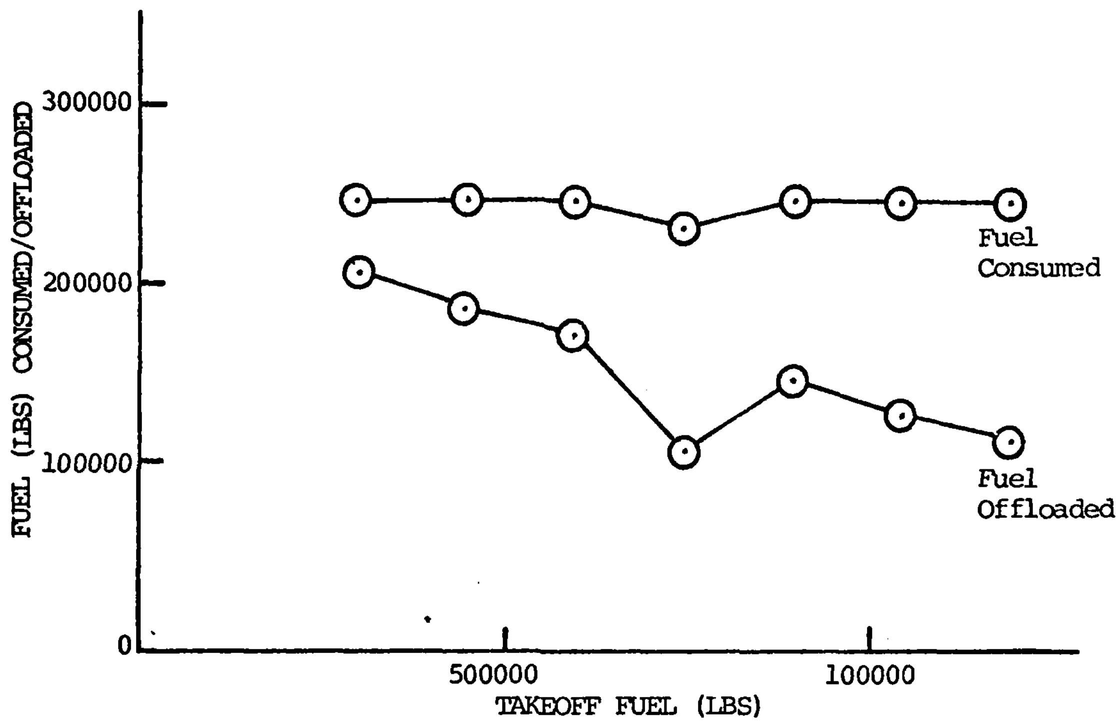


Fig. 12. C-141 Fuel Consumed/Offloaded
Cargo Load = 20,000 lbs

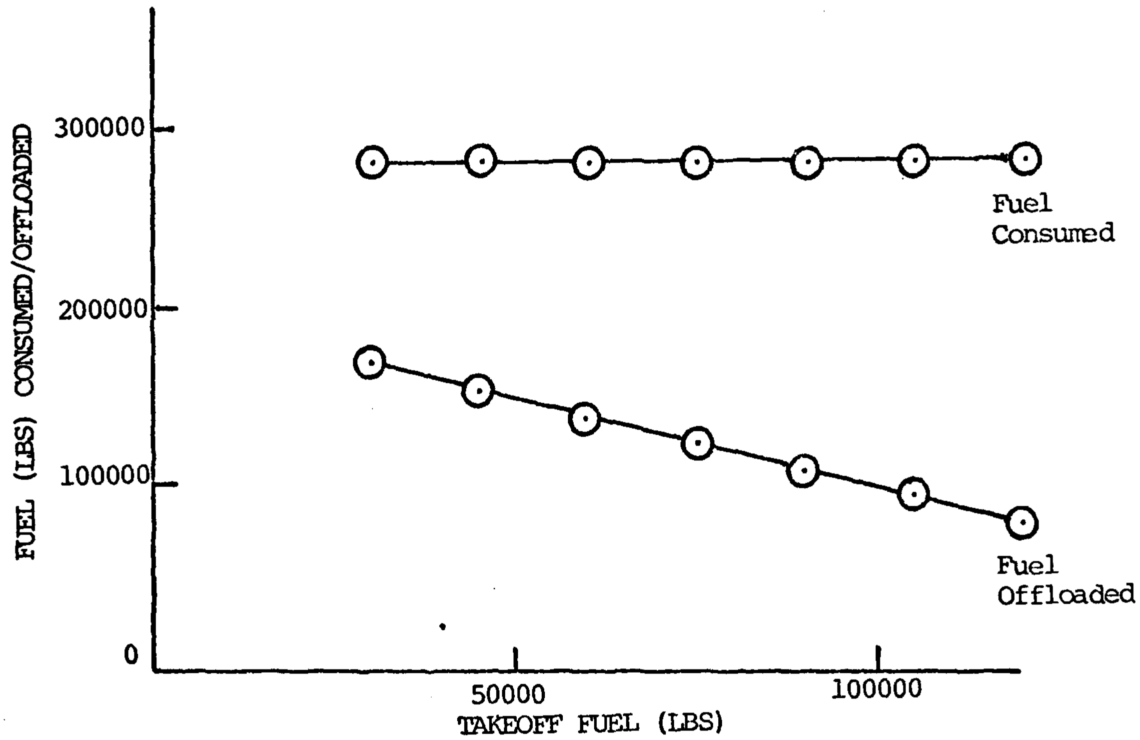


Fig. 13. C-141 Fuel Consumed/Offloaded
Cargo Load = 35,000 lbs

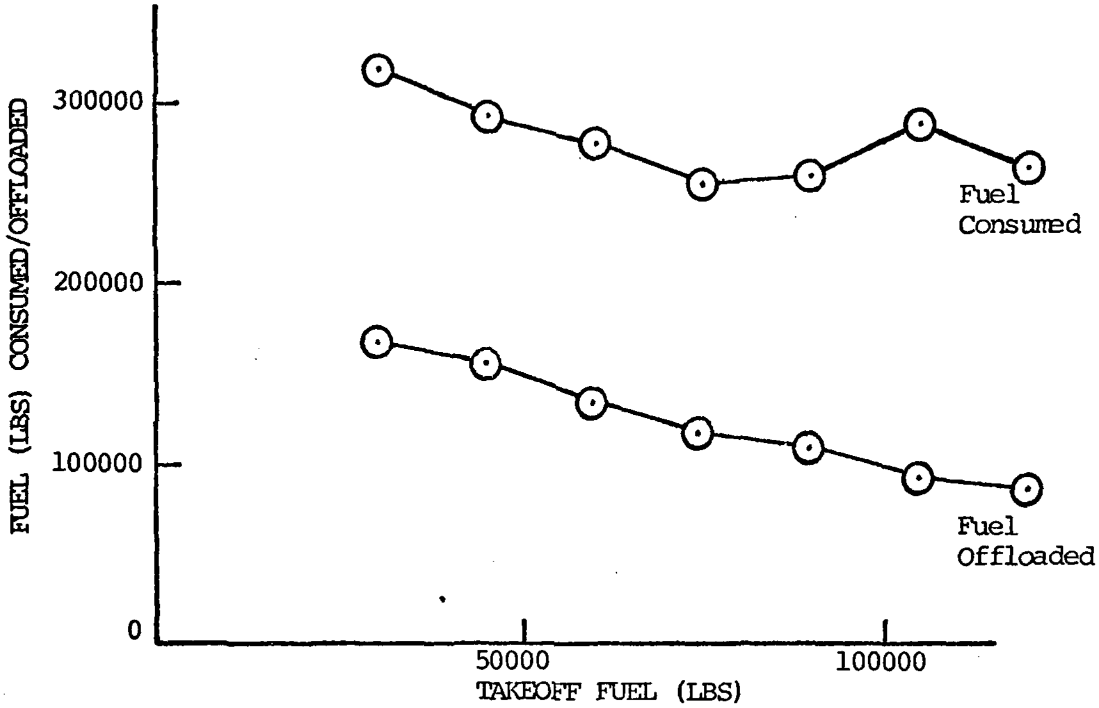


Fig. 14. C-141 Fuel Consumed/Offloaded
Cargo Load = 50,000 lbs

load had a much greater effect on the total fuel consumed than takeoff fuel. This result is due to the sensitivity of the airlifter's fuel consumption on total gross weight and, hence, cargo load. Changing the takeoff fuel weight many times just changes the amount of fuel offloaded at one or more aerial refueling points, which results in very small changes in total fuel consumed. Furthermore, the points chosen for AR also were not affected by takeoff fuel; the primary factor affecting the AR decision appears to be the distance of the nearest tanker base from the stage, which in turn affects the tanker's fuel consumption. Therefore, a stage with a tanker base nearby would have a much better chance of being used as an AR point than one with a tanker base far away.

A direct causal relationship between takeoff fuel and total fuel consumed does not appear to exist. In most of the combinations, total fuel consumed decreases as takeoff fuel increases, as might be expected. This is a result of the fact that more takeoff fuel in most cases decreases the need for AR. Figures 9, 11, 12, and 13 show cases where the total fuel consumed remains constant over a portion of the takeoff fuel range. This result is due to the model varying the fuel offload at the same AR point without using additional tankers. For example, in Figure 9, a takeoff fuel of 46,000 pounds results in a KC-10A fuel offload of 212,500 poinds at stage 1; as the takeoff fuel

increases to 196,000 pounds, the KC-10A offload decreases at stage 1 to a figure of 62,500 pounds. Since the aircraft gross weight remains the same in all cases, the airlifter's fuel consumption also does not change. The tanker's fuel consumption also stays the same, since the refueling requires one tanker in each case. Therefore, total fuel consumption remains constant. Figures 10, 12, and 14 show instances where the total fuel consumed increases as take-off fuel increases. This can be explained again by the fact that higher gross weights require more fuel; in these cases, the additional gross weight caused by the higher takeoff fuel increased fuel consumption to the point that an additional aerial refueling was required. This increased the response variable, total fuel consumed.

For the C-5A, Figures 9 through 11 graphically show the KC-135A to be more efficient than the KC-10A for the normal rendezvous mode of aerial refueling. This is because of the significantly higher fuel consumption of the KC-10A and the overall inefficiency of offloading all of the KC-10A's available fuel once it arrives at the AR point. Recent operational tests show the KC-10A most efficient in the "buddy" AR method, where the tanker accompanies the airlifter over its route of flight, refueling the airlifter as required. Using the KC-10A as a refueler/airlifter in this manner results in significant fuel savings by reducing airlifter requirements (by carrying up to three

C-141B or one C-5A cargo equivalent loads) and simultaneously reducing tanker requirements (by accomplishing all AR requirements along the way) (Ref 9).

Figures 9 through 14 also show the total fuel offloaded for various cargo loads and takeoff fuel weights. As is expected, fuel offload requirements decreased in most cases as takeoff fuel increased. Figure 12 shows one exception, where the offload requirement jumped to a higher value as takeoff fuel increased from 75,000 pounds to 90,000 pounds. This is one case where the higher gross weight (hence higher fuel consumption) caused an additional AR. In all other cases, the higher takeoff fuel effectively decreased AR requirements along the route of flight. For the C-5A, fuel offload requirements were always a little higher for the KC-10A than for the KC-135A. This is indirectly due to the higher fuel consumption of the KC-10A. Since more fuel was required to get the KC-10A to the AR point, this additional cost to the system forced higher offloads whenever the AR option was selected. These higher offloads increased airlifter fuel consumption, which in turn increased offload requirements later on.

<u>Part Two</u>

The objective of minimizing fuel consumption is clearly not always compatible with that of minimizing the use of tankers. Appendix G contains the number of tankers

used for each of the aircraft combinations in the experimental design. The use of tankers was minimized only to the extent of minimizing total fuel consumed. In many cases, the mission could be accomplished with fewer tankers by allowing the airlifter to offload more fuel from each tanker, up to a maximum of FTNK. This, of course, increases the fuel consumed by the airlifter and, hence, by the total system.

From the results of the data used in the analysis, the real benefits derived from the use of the AR optimization model are modified slightly into two main areas. First, as mentioned in Chapter I, its use during peacetime operational mission planning will result in a considerable fuel savings where the number of tankers used is not a critical factor. In fact, using more tankers increases peacetime training missions without additional fuel costs. This is important to the Strategic Air Command, where aircrew members have traditionally been shorted flying hours due to fuel costs. Second, the model can be used during planning for large scale airlift operations, where a number of airlifters are enroute to a common destination via the same route (these are known as airlifter enroute cells). Unused tanker offload capability after one aerial refueling can be used to refuel a subsequent airlifter. This type of operation would bring together the two objectives of minimizing tanker use and fuel consumed. This type of

analysis is also what is needed to determine tanker force requirements for a given contingency.

Part Three

Appendix G presents the scenario used and the data obtained by varying the number of iterations used in the optimization (ITMAX) and the fuel state step size (NSTEP). It was thought that these two parameters had the most effect on the efficiency of the Box' algorithm. Other model parameters were left at the values recommended by Box (Ref 6).

Varying ITMAX had no effect on the results of the model output, at least for this scenario. As might be expected, the convergence criteria stopped the iteration process as soon as the complex points' functional values remained within .01 of each other for five consecutive iterations. When ITMAX was increased beyond 100, the computer execution time remained constant, as did the total fuel consumed figure. This indicates that the algorithm was behaving as intended.

As NSTEP was decreased, the step size between allowable fuel states increased, and the optimal solution for total fuel consumed changed slightly. Decision values only changed in the third decimal place, and the optimal aerial refueling points stayed the same. The changes in fuel offload and total fuel consumed are small as compared

to the actual figure. As might be expected, the parameter NSTEP only tends to refine the model output and can be varied by the user as needed to decrease computer execution time.

Small values of NSTEP (less than 50) can generate errors of 2 percent or greater between consecutive stage fuel states. This is because, when calling back the optimal solution after recursion, the model tries to match state values between stages to recover the next stage optimal solution. If it cannot match the state values exactly, it selects the next higher state value and its optimal solution, which is at most 1/NSTEP in error. Therefore, NSTEP = 100 reduces the error to 1 percent, and this value was selected for all data runs in this study. In general, the higher NSTEP is, the more accurate the model output will be; however, computer execution time increases in direct proportion to NSTEP.

On two data runs with NSTEP equal to 100, the model forced an AR with a small offload of less than the state step size. This was also caused by the model's inability to exactly match state values during the optimal solution call-back. This situation can be corrected by increasing NSTEP to 150--the model then combines this small offload with another AR later along the route of flight. In both cases, the optimal solution was not affected by more than 1 percent.

Part Four

Six typical airlift missions were selected for use in comparing model output with actual computer flight plan data. These missions, input data, model output, computer flight plan information, and comments on each comparison are presented in Appendix G. This section contains general comments on the comparisons.

Present computer flight planning procedures appear extremely effective in using optimal refueling points, since model selected AR points generally agree with those used on actual missions. From the six examples selected for analysis, it appears that fuel is wasted by offloading too much fuel at each AR point. This results in higher fuel consumption (approximately 25 percent of extra fuel carried is burned just by carrying it) and excessive fuel loads overhead destination. Model fuel states are in close agreement with those on the computer flight plans, verifying the fuel consumption equations used in the model. The small differences here can be attributed to one or both of the following:

1. The model assumes great circle distances between stage points. The computer flight plan uses airway or route distances between navigation aids as published on ICAO aeronautical charts. The computer flight plan will report larger fuel consumption figures if this is the situation.

2. The model assumes constant wind factors over each segment between stages, where the computer flight plan changes the wind factor as necessary at each navigation aid or intersection between stages. This will have a varying effect on fuel consumption.

In each case, model fuel consumption for the airlifter was less than that for the computer flight plan. This is primarily due to the computer flight plan carrying too much fuel over the entire route. Reference to the data in Appendix G shows this to be the case in each of the sample airlift scenarios.

The question of isomorphism--how true must the model be to the actual system--is now addressed. This model is not a mission fuel planning tool like the computer flight plan; therefore, it is felt that fuel computations need not be as accurate as those from the computer flight plan. They should, however, be as responsive to gross weight changes as in the real aircraft. They also should be accurate enough so they do not effect the optimization procedure. These example scenarios, plus the previous analyses, show that the model meets these requirements. The model is a reasonable representation of the real system and uses a proven optimization technique; therefore, the user can feel confident that the model is performing as intended.

V Conclusions and Recommendations

The primary objective of this research was to develop an operationally usable aerial refueling optimization model. The various subobjectives were organized in an effort to verify and validate the results of the model. The overall conclusion of the analysis presented in Chapter IV is that the AR optimization model found in Appendix C performs as intended. Using dynamic programming techniques, the model determines optimal aerial refueling points, fuel offloads, and tanker departure bases, for any airlifter mission scenario. This conclusion is based on the following general observations:

1. Dynamic programming is a proven solution technique for a problem of this type. The Box optimization procedure was also verified using a sample problem, the results of which are in Appendix F.

2. The results of the experimental design and analysis are reasonable and easily verified. Response variables varied as expected, from the system analysis done in Chapter II, as the control variables were changed.

3. Model output shows reasonable agreement with actual computer flight plan data. Fuel consumption models performed as intended. Also, model-selected aerial

refueling points were surprisingly close to those actually used on operational AR missions.

4. The model shows a high degree of isomorphism with the real system.

In addition to verifying and validating the model, the experimental design served to answer some questions on the aerial refueling problem:

1. For the scenario used in the study, cargo load statistically had more effect on total fuel consumed than takeoff fuel.

2. There is no general, direct relationship between takeoff fuel and total fuel consumed. The total fuel consumed is a complex function of takeoff fuel, tanker base location, airlifter route of flight, and diversion base location. All factors interact to determine the optimal AR points and tanker offloads. In most cases, however, higher takeoff fuel weights result in lower total fuel consumed figures (for this scenario).

3. For the scenario used in the analysis, the KC-135A is more efficient than the KC-10A in the normal rendezvous mode of aerial refueling.

4. The objective of minimizing total fuel consumed is not always compatible with that of minimizing the use of tankers. Nevertheless, the use of the model can be highly beneficial during peacetime operational mission planning and contingency tanker force determination.

5. The location of tanker departure bases has a much greater effect on the optimal aerial refueling point than originally expected. This fact alone may have some impact on future aerial refueling mission planning.

Recommendations

The following recommendations are submitted as a result of this research:

1. The planning of strategic airlift aerial refueling missions should include the use of this model to determine optimal rendezvous points and fuel offloads. This will result in significant fuel savings to the Air Force, and will maximize training per flight hour for both MAC and SAC.

2. Model output should be used during the operational planning phase of contingency planning. It can be used to determine tanker force requirements for a contingency, allowing for more efficient tanker coordination between MAC and SAC.

Suggestions for further research in this area include the following:

1. To be more beneficial, the model should somehow handle the incompatibility of minimizing total fuel consumed and the use of tankers. Reformulation of the problem to consider both objectives would allow the model to be useful in studying overall tanker force requirements.

2. Decreasing the fuel state step size (that is, increasing NSTEP) would result in a slightly more accurate model. Also, increasing the number of stages, or points defining the airlifter's route of flight, would result in output more sensitive to the airlifter's actual flight path. Making the model computationally more efficient may allow the user to do both of these without increasing computer execution time beyond a practical limit.

3. The "buddy" rendezvous method of aerial refueling should be investigated and compared with the results of this study. The results of this study and one on the "buddy" method could be used to come up with an optimal combination of airlifters and tankers.

4. Wind factor data should be used when calculating tanker enroute times. Furthermore, a historical wind factor data base could be incorporated into the model, based on season and geographic location. This data could be used anywhere in the model whenever a wind factor is required and would avoid the user having to input this data.

5. The model could be modified to handle airlifter enroute cells. This method of airlifter employment may prove to be the most efficient in a large scale operation.

6. The model could be modified to allow tankers to depart with only enough fuel for the mission, rather than maximum fuel load. This study would result in further fuel savings. Also, making the tanker fuel flows dependent on

actual tanker gross weight rather than an average gross weight would further refine the model.

7. Finally, using actual computer flight plan distances for both airlifter and tanker routes of flight would increase model accuracy.

Bibliography

1. _Airlift Operations School Supplemental Readings Text_, AOS Director of Curriculum, AOS/EDC, Scott AFB, Ill., January 1981.

2. Baker, Kenneth R. and Linus E. Schrage. "Finding an Optimal Sequence by Dynamic Programming: An Extension to Precedence-Related Tasks," _Operations Research_, _26_: 111-119 (January-February 1978).

3. Bellman, Richard E. III. _Dynamic Programming_. Princeton, N.J.: Princeton University Press, 1957.

4. Bent, Dale H., _et al_. _Statistical Package for the Social Sciences_. New York: McGraw Hill, Inc., 1975.

5. Bordelon, Vernon P. and John C. Marcotte. "Optimization of Strategic Airlift In-Flight Refueling." Unpublished M.S. thesis. School of Engineering, Air Force Institute of Technology, Wright-Patterson AFB, Ohio, March 1981.

6. Box, M. J. "A New Method of Constrained Optimization and Comparison With Other Methods," _The Computer Journal_, _8_: 42-51 (April 1965-January 1966).

7. _C-141B Performance Manual_. T.O. 1C-141B-1-1, U.S. Government Printing Office, Washington D.C., 27 March 1980.

8. _C-5A Performance Manual_. T.O. 1C-5A-1-1, U.S. Government Printing Office, Washington D.C., 1 December 1977.

9. Castledine, Susan. "Air Force Deploys KC-10; Savings Cited," _Aviation Week and Space Technology_, _114_: 64-76 (15 June 1981).

10. "Defense Budget Cuts Proposed to Congress," _Aviation Week and Space Technology_, _115_: 18-20 (28 September 1981).

11. Fishman, G. S. and P. J. Kiviat. "The Analysis of Simulation--Generated Time Series," _Management Science_, Vol. 13, No. 7, March 1967.

12. Gosch, W. D. and W. F. Mooz. "A USAF Energy Consumption Projection Model." Report prepared for the Defense Advanced Research Projects Agency. November 1974.

13. Harp, Tillford, Captain, USAF. HQ MAC/XPSR (personal interview). Scott AFB, Ill., 16 June 1981.

14. Hillier, Fredrick S. and Gerald J. Lieberman. Introduction to Operations Research. San Francisco: Holden-Day, Inc., 1980.

15. KC-135A Performance Manual. T.O. 1KC-135A-1-1, U.S. Government Printing Office, Washington D.C., 1 December 1977.

16. Kuester, James L. and Joe H. Mize. Optimization Techniques With Fortran. New York: McGraw Hill Book Company, 1977.

17. M-14 Baseline Model Operation--SHEA Study (Phase I), Director, Studies and Analysis, HQ MAC/DCS Plans, Scott AFB, Ill., 18 September 1980.

18. MACR 55-2/55-141, Strategic Airlift Operations (C-5A and C-141). Department of the Air Force, HQ MAC, Scott AFB, Ill., January 1981.

19. MACR 55-20/55-21, Fuel Planning Document (C-141B and C-5A). Department of the Air Force, HQ MAC, Scott AFB, Ill., January 1977.

20. Milton, T. R. "The Aerial Tanker and Maritime Strategy," Air Force Magazine, 64: 87 (July 1981).

21. Shannon, Robert E. Systems Simulation--The Art and Science. Englewood Cliffs, N.J.: Prentice Hall, Inc., 1975.

Appendix A

Fuel Consumption Models

The general model used in deriving an aircraft fuel flow equation is the following:

$$\hat{FF} = b_o(TAS)^{b_1}(GW)^{b_2} + \varepsilon_i \qquad (14)$$

where b_o, b_1, and b_2 are constants, TAS is the aircraft true airspeed in knots, GW is the gross weight in pounds, and $\hat{FF}$ is the estimated fuel flow in gallons per hour. This equation can be reduced to the general log-linear form:

$$\ln(\hat{FF}) = \ln(b_o) + b_1 \ln(TAS) + b_2 \ln(GW) + \varepsilon_i \qquad (15)$$

from which a multiple linear regression can be performed. Reference 12 uses this procedure for determining a general fuel consumption model for all Air Force aircraft collectively; however, this appendix will apply this procedure to refine the equation for use in the AR optimization model.

Table A-1 summarizes the data used in the regression. One model was derived for both airlifters (C-5A and C-141B), and one for each of the tankers (KC-10A, KC-135A). Aircraft gross weights were selected throughout the operating range of each aircraft, and true airspeeds were calculated based on the optimum altitude capability of each aircraft as a function of gross weight. Therefore, the data used assumes that each aircraft flies as close as possible to its optimum altitude based on its present gross

TABLE A-1

DATA USED FOR FUEL FLOW REGRESSION

KC-135A				
GW (lb)	Opt Alt	TAS (kt)	FF(lb/hr)	FF(gal/hr)
120,000	45,000	470	7121	1096
140,000	42,000	470	8246	1268
160,000	39,000	470	9216	1418
180,000	36,500	470	10444	1607
200,000	34,000	473	11537	1775
220,000	32,000	477	12892	1983
240,000	29,000	485	14265	2195
260,000	26,000	491	15839	2437
270,000	24,000	495	17067	2626
C-141B/C-5A				
GW (lb)	Opt Alt	TAS (kt)	FF(lb/hr)	FF (gal/hr)
190,000	44,800	433	7600	1169
210,000	42,700	433	9000	1385
230,000	40,800	433	10200	1569
250,000	39,200	433	11000	1692
270,000	37,500	433	11800	1815
290,000	35,900	433	12800	1969
310,000	34,300	436	13800	2123
330,000	32,800	440	14000	2154
400,000	41,500	443	14800	2277
450,000	39,500	443	16720	2572
500,000	37,000	443	18320	2818
550,000	35,000	445	20160	3101
600,000	33,000	448	22400	3446
650,000	29,000	456	24800	3815
700,000	27,500	468	27120	4172

weight. This is what is done in practice whenever possible. Also, the temperature deviation from standard was assumed to be +10 degrees Centigrade, which again is a typical situation. The fuel flow for each data point was obtained from the applicable aircraft performance manual in pounds per hour, and converted to gallons per hour using the conversion factor of 6.5 pounds per gallon. C-5A fuel flow data alone were used in deriving the KC-10A fuel flow model; Reference 5 presents an analysis justifying the use of C-5A data to estimate KC-10A fuel consumption.

Regression Results

The Statistical Package for the Social Sciences (SPSS) multiple linear regression routine was used in all three cases, resulting in the following fuel flow models:

$$\text{Airlifters: } \ln(FF) = -6.3126 + .8344\ln(GW) + .5503\ln(TAS),$$

$$\text{or} \quad FF = 1.81\times10^{-3}(GW)^{.8345}(TAS)^{.5503} \qquad (16)$$

$$\text{KC-10A: } \ln(FF) = -11.7366 + .9795\ln(GW) + 1.1214\ln(TAS),$$

$$\text{or} \quad FF = 7.995\times10^{-6}(GW)^{.9795}(TAS)^{1.1214} \qquad (17)$$

$$\text{KC-135A: } \ln(FF) = -17.4898 + .9161\ln(GW) + 2.239\ln(TAS),$$

$$\text{or} \quad FF = 2.54\times10^{-8}(GW)^{.9161}(TAS)^{2.239} \qquad (18)$$

The SPSS Summary Table for the airlifter fuel flow model is shown in Table A-2, and the predicted and actual fuel flows are plotted in Figure A-1. Statistically, the

TABLE A-2

SPSS SUMMARY TABLE--AIRLIFTER

	TAS	GW
F to Enter or Remove	.13615	112.97
Significance	.719	.000
Multiple R	.889	.990
R Square	.791	.979
R Square Change	.791	.189
Simple R	.89	.99
Overall F	292.5	
Significance	.000	

model is highly significant, with a coefficient of determination (R^2) value of .979. This indicates that 98 percent of the variability in fuel flow is explained by the regression model. The high overall F value of 292.50 would cause a rejection of the hypothesis H_o, where

$$H_o: \beta_1 = \beta_2 = 0$$

H_1: at least one inequality

confirming that true airspeed and gross weight together contribute significant explanatory power to the regression model. In a fuel consumption estimating sense, Figure A-1 shows reasonable agreement between actual and predicted fuel flows.

Table A-3 contains the Summary Table for the KC-135A model, and Figure A-2 shows a comparison between actual and

TABLE A-3

SPSS SUMMARY TABLE--KC-135A

	TAS	GW
F to Enter or Remove	46.25	1446.73
Significance	.000	.000
Multiple R	.897	.999
R Square	.805	.999
R Square Change	.805	.194
Simple R	.897	.996
Overall F	3712.58	
Significance	.000	

predicted fuel flows. Here, R^2 is .99, indicating that the model is able to explain 99 percent of the variability in fuel flow. The extremely high F value of 3712 confirms the result as a highly significant model in a statistical sense. Figure A-2 shows extremely good agreement between actual and predicted fuel flow values.

The Summary Table for the KC-10A model is given in Table A-4, also indicating a highly significant model. Again, 99 percent of the variability in fuel flow is explained by the model. Figure A-3 shows good agreement once again between predicted and actual values.

TABLE A-4

SPSS SUMMARY TABLE--KC-10A

	TAS	GW
F to Enter or Remove	16.85	1207.16
Significance	.015	.000
Multiple R	.861	.999
R Square	.741	.999
R Square Change	.741	.258
Simple R	.861	.998
Overall F	2339.43	
Significance	.000	

<u>Validity of the Model</u>

$$Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon_i \qquad (19)$$

Given the above general model, the linear regression procedure assumes the following to be true:

1. It assumes that the mean of Y, given X_1, X_2, is predicted by the above multiple linear regression model.

2. The error term, ε_i, is a random variable, normally distributed, with its mean equal to zero.

3. ε_i also has a constant variance.

All residuals were tested by the SPSS routine and found to be within two standard deviations of the mean. Although this test does not confirm normality of the residuals, it

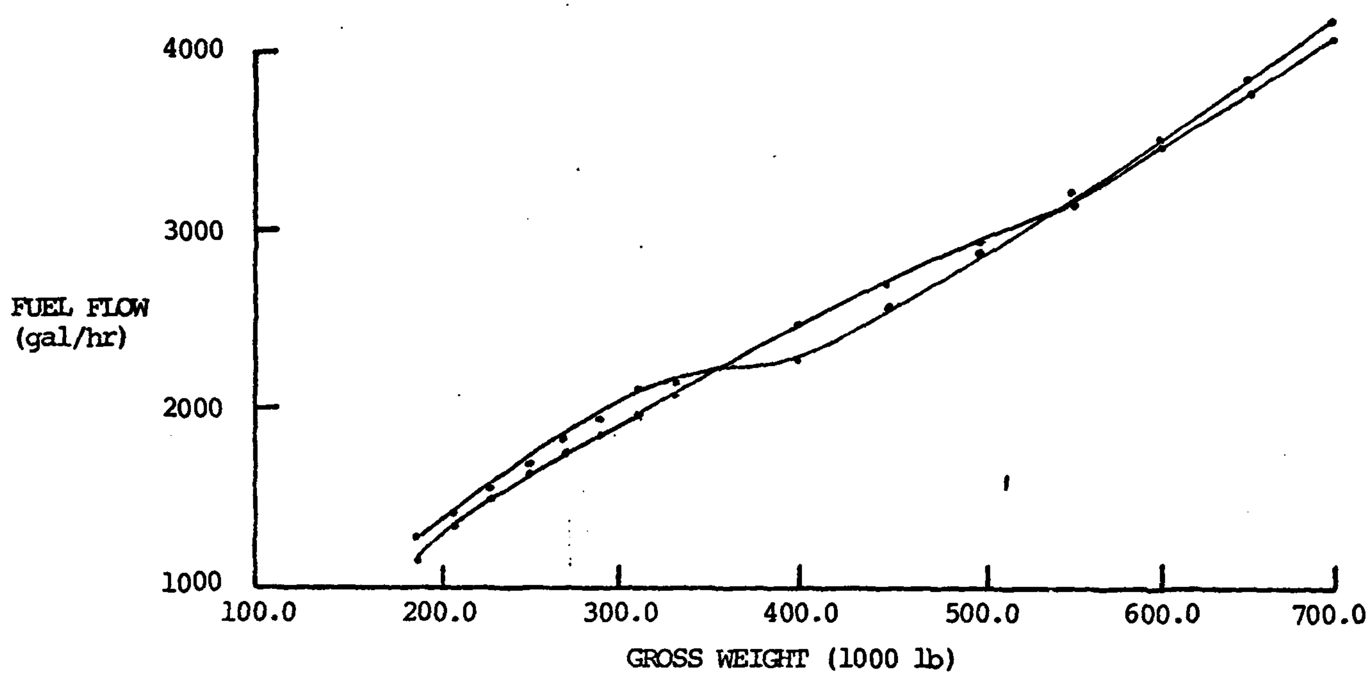


Fig. A-1. Comparison of Fuel Consumption Model With Actual Data--Airlifters

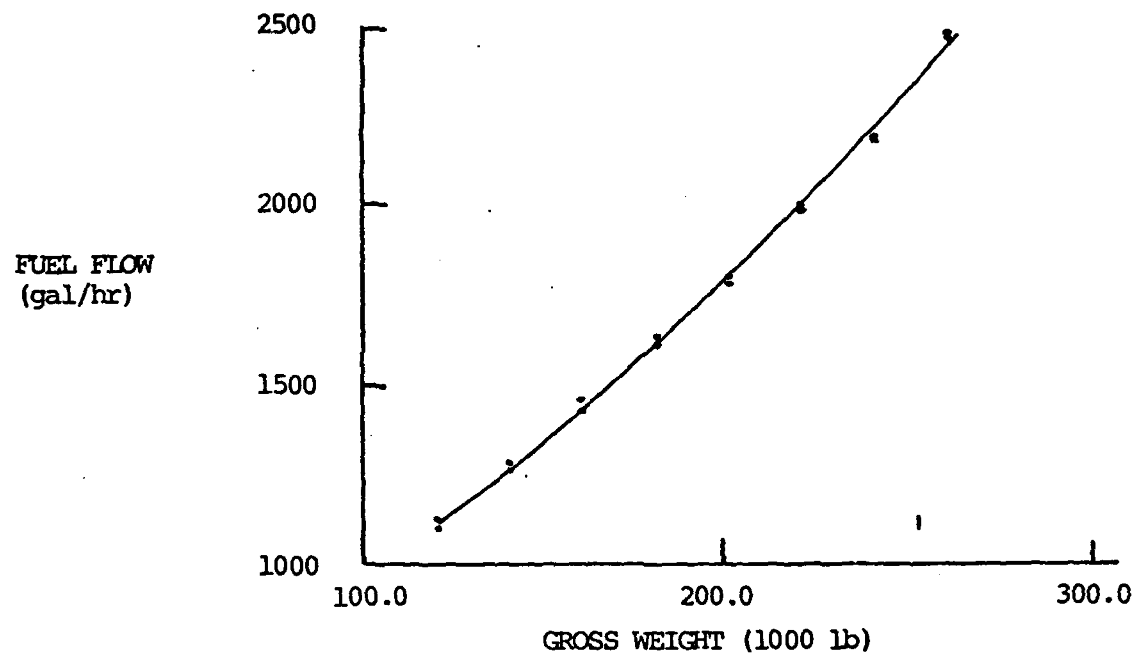


Fig. A-2. Comparison of Fuel Consumption Model With Actual Data--KC-135A

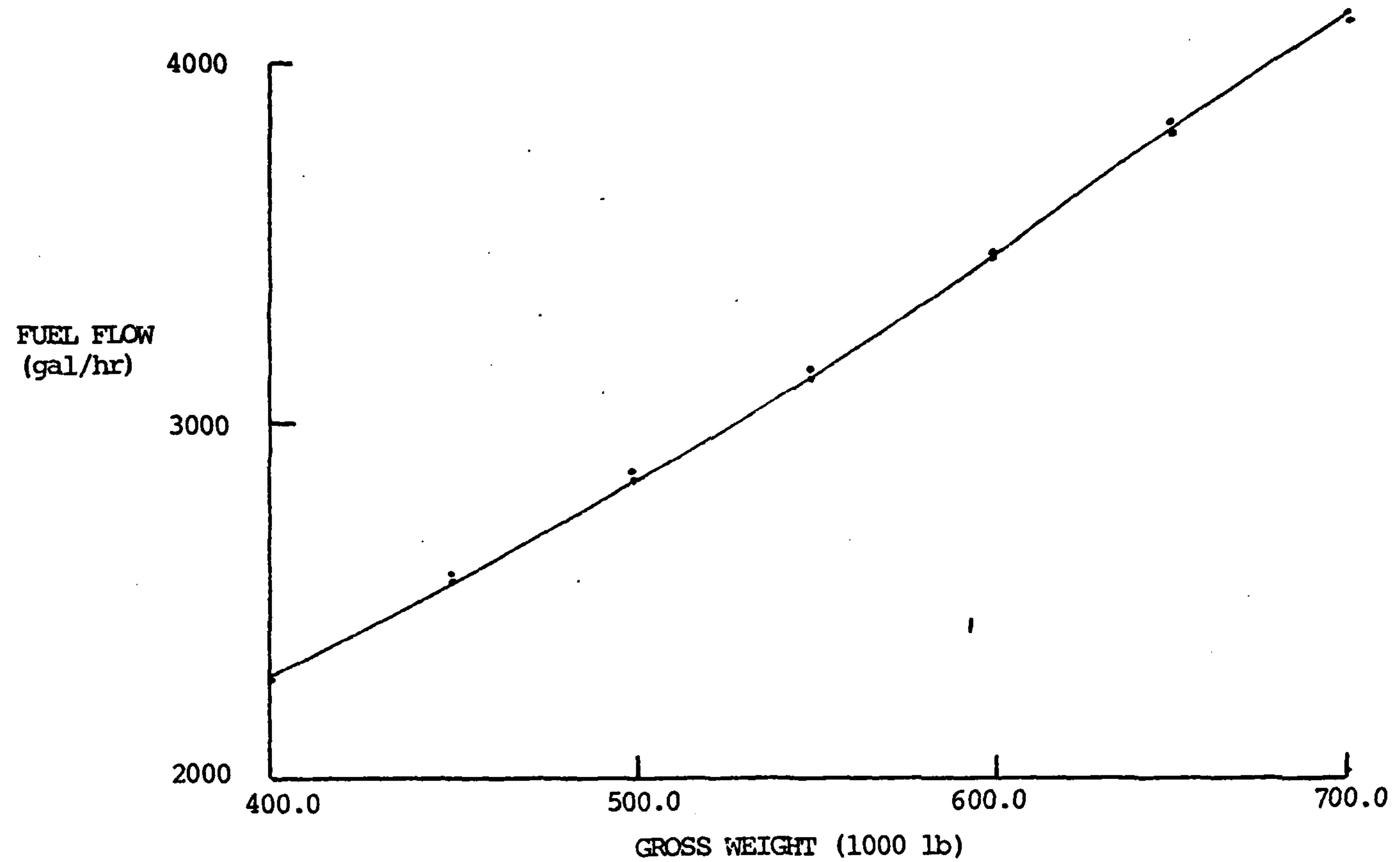


Fig. A-3. Comparison of Fuel Consumption Model With Actual Data--KC-10A

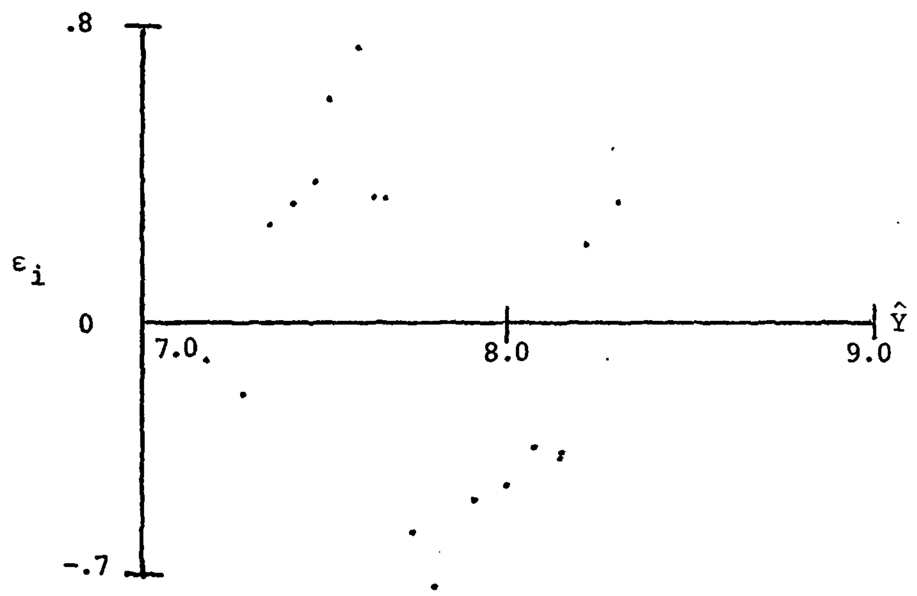


Fig. A-4. Residual Plot--Airlifters

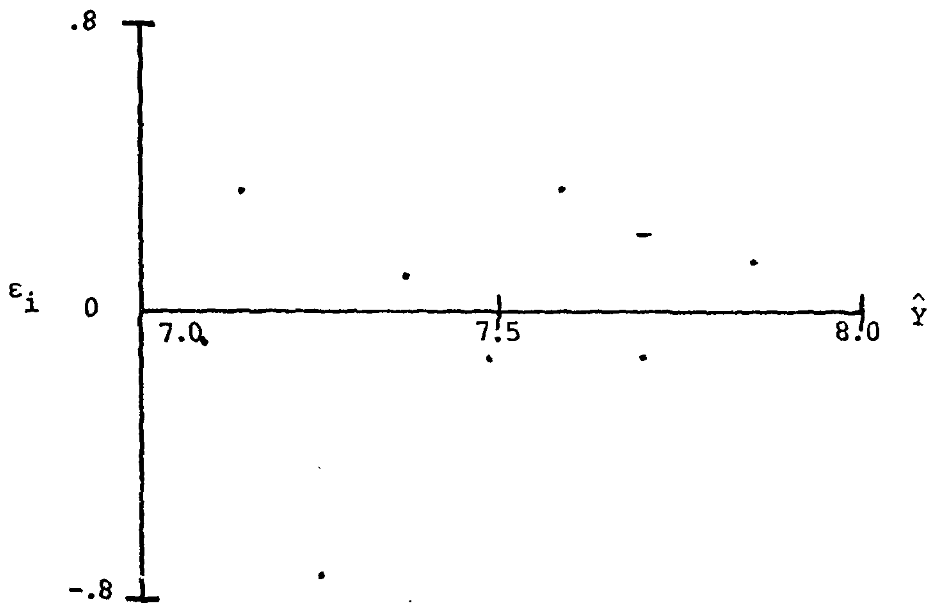


Fig. A-5. Residual Plot--KC-135A

AD-A115 699   AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOO--ETC F/G 15/7
AN AERIAL REFUELING OPTIMIZATION MODEL APPLIED TO STRATEGIC AIR--ETC(U)
MAR 82 T A LINDHOLM

UNCLASSIFIED AFIT/GST/OS/82M-7 NL

2 OF 2

AD A
115699

END
DATE
FILMED
07-82
DTIC

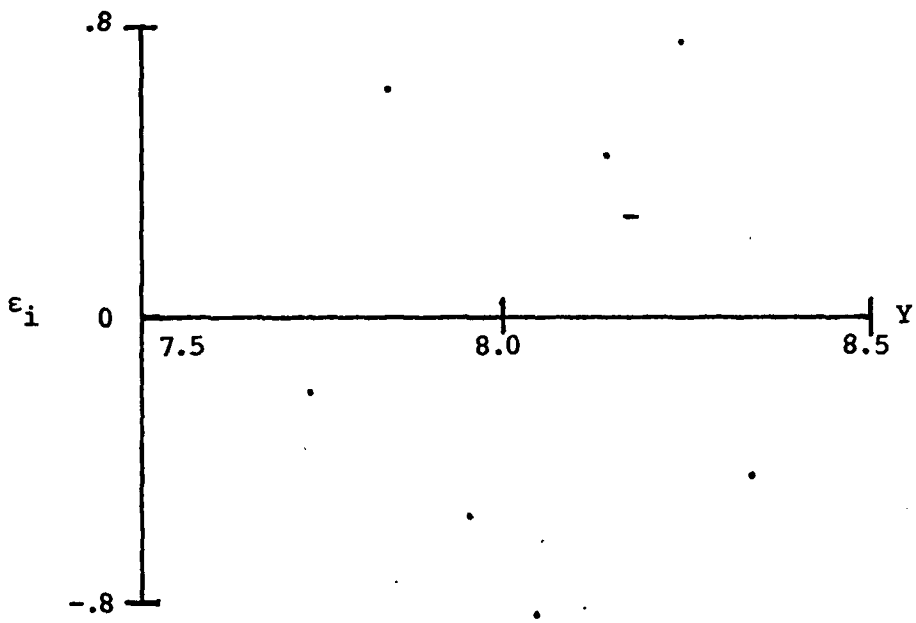


Fig. A-6. Residual Plot--KC-10A

does indicate a lack of evidence against the normality assumption; therefore, it is assumed that the second assumption is a valid one. The constant variance assumption of the error term, ε_i, was checked by plotting the residuals against FF, the predicted fuel flow values. These plots are shown in Figures A-4, A-5, and A-6. In each case, the residual appears to be independent of the change in the predicted fuel flow; therefore, the constant variance assumption is a valid one.

The variance/covariance matrix (as determined by SPSS) for each of the three fuel flow models indicates that the coefficients as determined by the regression are essentially independent. The small covariance between variables further substantiates the validity of the models.

Appendix B

Program DYNAM Description

This section explains in detail the computer code for the AR optimization model contained in Appendix C. Figure B-1 contains a logic diagram showing how the main program DYNAM interacts with the various subroutines. A separate appendix (Appendix D) is devoted to explaining the Box "Complex" algorithm.

The program consists of a main program (DYNAM), three general subroutines (COMPLX, CHECK, and CENTR), and seven other subroutines designed specifically for the AR optimization model (FUNC, CONST, SCONST, RETURS, TRANS, DISXRT, and DISBAS). The main program coordinates the subroutines to provide the stage optimums and performs the recursion to recover the optimum path. Final returns, decision values, and state values are printed from DYNAM; intermediate results can also be printed from subroutine COMPLX if the user desires. The main program and three general subroutines were obtained from Reference 16, with some modification. All subroutines were specifically written for the AR optimization problem.

DYNAM has several user-selectable options for printing results and formulating the problem. The parameter IPRINT selects the printing option to be used according to the following:

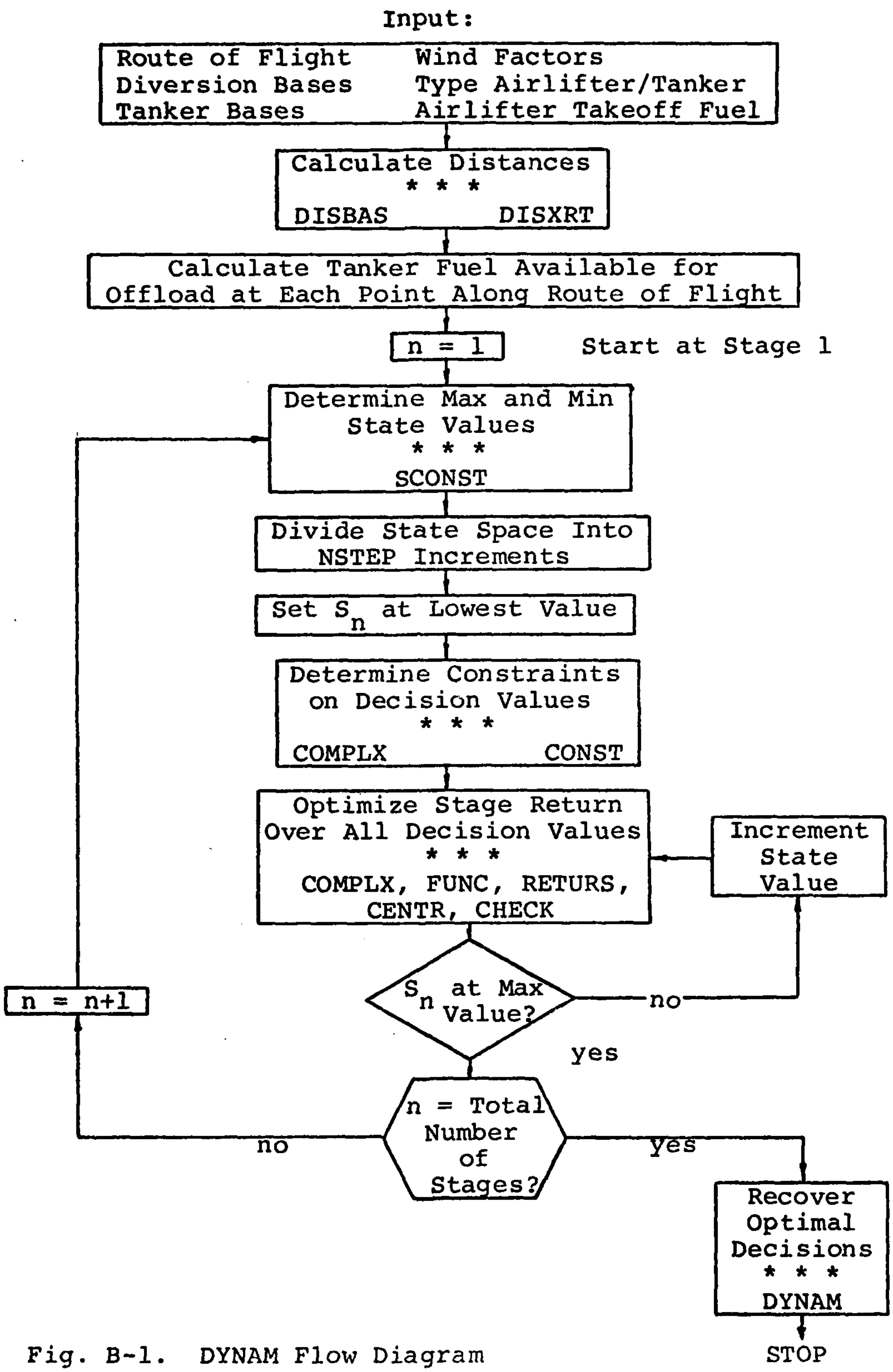


Fig. B-1. DYNAM Flow Diagram

IPRINT = 0: Only print final results.

IPRINT = 1: All search values and tables are printed.

IPRINT = 2: State, return, and decision values are printed after each stage is optimized.

The parameter IPROB selects the type of problem to be solved:

IPROB = 1: Initial value problem; given a takeoff fuel weight, the program will optimize stage decisions without regard to the overhead destination fuel.

IPROB = 2: Final value problem; given a required overhead destination fuel, the program will optimize stage decisions and will determine takeoff fuel weight.

IPROB = 3: Initial value/final value problem; given both takeoff fuel and required overhead destination fuel, the program determines the optimal stage decisions. This option is used in this analysis.

DYNAM also has the capability to optimize an objective function over more than one decision at each stage; for example, the model could be modified to determine the optimal type of tanker and number of tankers at each stage. The parameter NDECIS defines the number of types of decisions at each stage. To handle this modification, however, dimension statements and common blocks would have to be changed (see Reference 16).

The following discussion describes each of the unique subroutines in detail.

Subroutine SCONST

Subroutine SCONST is called by the main program before the recursive analysis of each stage to set the minimum and maximum allowable fuel states allowed at each stage. In each case, these constraints apply to the airlifter's fuel state prior to any refueling done at that stage.

Minimum State Bound. For stage 1 (overhead destination), the lower bound on the state value is determined by the required overhead destination fuel figure. For all other stages, the subroutine calculates a fuel requirement for the airlifter to divert to the nearest diversion base from that stage (SCON1). It also calculates the fuel required for the airlifter to fly the next portion of its route of flight (SCON2). SCONST then selects the lowest of these two fuel figures to be the state lower bound at that stage. This insures that the airlifter will have enough fuel to either divert to a suitable alternate or continue to the next stage if refueling is either unsuccessful or not accomplished.

Maximum State Bound. For stage 1, the upper bound on the state value is again the required overhead destination fuel. This forces the main program (DYNAM) to come as close as possible to this fuel state during the recursive calls to recover the optimal system solutions, thereby

discouraging the offload of excess fuel. The lower bound acts like a hard constraint, whereas the upper bound does not. For the remaining stages, the subroutine selects the state value as constrained by the airlifter's maximum fuel capacity or maximum allowable gross weight, whichever is more restrictive.

The main program then, using the state value constraints, divides the allowable state space into 100 increments (as defined by the parameter NSTEP). Starting with stage 1 and the lowest possible state value, DYNAM coordinates the rest of the subroutines to determine an optimal decision value, or number of tankers to be offloaded, for each of the 100 possible state values. This is done to minimize the return function for each stage, and the optimal decision values and returns for each possible state are stored for recall when the system optimal solution is recovered by DYNAM.

Subroutine CONST

Again, the general problem is to

$$\begin{aligned} &\text{Minimize} \quad F(r_1, r_2, \ldots, r_n) \\ &\text{Subject To} \quad G_i \le d_i(S_i) \le H_i \qquad (20) \end{aligned}$$

The subroutine CONST determines the decision value constraints as a function of the fuel offload requirements. The decision value for a stage represents the number of

tankers offloaded at that stage. Therefore, the constraints are defined in CONST as fractional parts of the tanker's available fuel for offload, or

$$G_i = \text{(Fuel Requirement)}/FTNK_i \tag{21}$$

where G_i is the lower constraint, and $FTNK_i$ is the maximum amount of fuel one tanker can offload at stage i.

Lower Constraint. CONST first calculates the fuel required by the airlifter to fly to the next stage (F1). It then calculates the fuel required to fly from the next stage to the closest diversion base (F2). If the sum of F1 and F2 is greater than the state value presently being considered, the lower bound becomes

$$[F1+F2-\text{(STATE VALUE)}]/FTNK \tag{22}$$

If not, the lower bound is zero. This insures that if the state value presently under consideration will not allow flight to the next stage and diversion at that stage (if AR is unsuccessful), the airlifter will be forced to refuel to at least meet this requirement. For stage 1, the lower constraint is set to zero, since it would be meaningless to refuel overhead destination.

Upper Constraint. The upper bound on the decision variable is also set by either the maximum fuel capacity,

or the maximum gross weight of the airlifter, whichever is more restrictive:

$$H_i = (XMAX-SLOW)/FTNK_i \tag{23}$$

where XMAX is either the maximum fuel capacity or the maximum fuel weight as limited by maximum gross weight, and SLOW is the lowest possible state value. For stage 1, the upper bound is also set to zero.

If, for a particular stage, the tanker bases are so far away that FTNK is less than zero, CONST returns a value for G_i and H_i of zero. This eliminates any possibility of refueling at a point which is infeasible and forces the recursive analysis to look for fuel elsewhere in the system.

These bounds on the decision values are used by COMPLX (see Appendix D) to select an initial set of feasible decision values for the recursive optimization procedure. Subroutine CHECK is used by COMPLX to keep all possible decision values during each iteration within these limits.

<u>Subroutine</u> <u>RETURS</u>

Subroutine RETURS calculates the total return for each stage as a function of the state value and decision value under consideration. The stage return is defined as the fuel required by the tanker(s) to fly from departure

base to the stage and return, plus the fuel required by the airlifter to fly from that stage to the next one along its route of flight. RETURS is called by subroutine FUNC each time either the stage, state, or decision value changes during the iterative process. The return for each combination at each stage is used by COMPLX during the optimization procedure, since the objective function is the sum of the individual stage returns:

$$\text{Minimize} \quad F(r_1, r_2, \ldots, r_n) = \sum_{i=1}^{n} r_i \qquad (24)$$

Subroutine TRANS

Subroutine TRANS is called by DYNAM, COMPLX, and FUNC any time a state transformation is required between stages. TRANS contains the transition function for the dynamic programming procedure. The subroutine begins with the fuel state at stage i+1, subtracts the fuel required by the airlifter to fly to stage i, and adds the fuel onload at stage i+1 to determine the fuel state at stage i. The transition is a function of the state values and decision values under consideration.

Subroutine FUNC

This subroutine formulates the objective function for each state value before the optimization over all decision values process begins. COMPLX is the primary

user of subroutine FUNC, and FUNC itself calls RETURS and TRANS while determining the objective function. For each stage, COMPLX calls FUNC with the state value under consideration and its associated decision values as inputs, from which a return is calculated for each decision using RETURS. FUNC then uses subroutine TRANS and the state value to calculate the corresponding state of stage n-1. Then, FUNC searches through the table of state values for stage n-1 until it matches the calculated state value from TRANS with one of these stored state values. Since the stored state value has an optimal decision and function value associated with it, based on the recursive analysis up to that point, the stored function value for stage n-1 is added to the return calculated from RETURS to form the new function value. The sum represents the objective function for stage n given the state value under consideration, and is the function to be minimized. This is done each time the decision value is changed during the optimization procedure in COMPLX.

Subroutine DISBAS

Subroutine DISBAS calculates the great circle distance in nautical miles between two points using geographic coordinates. The following expression is used to calculate the angular difference in radians:

$$D = \cos^{-1}[\sin(MACLA)*\sin(XP1)+\cos(MACLA)*\cos(XP1) *\cos(abs(XP2-MACLO))] \quad \text{(Ref 5:168)} \qquad (25)$$

where

MACLA = Latitude of the stage

MACLO = Longitude of the stage

XP1 = Latitude of the tanker departure base

XP2 = Longitude of the tanker departure base

The distance in nautical miles is then computed by using the following relationships:

1. One degree of latitude = 60 nautical miles.
2. One radian = 57.2958 degrees.

DISBAS computes the distances between the stage and all tanker departure bases included as input. It then selects the base with the smallest distance, stores the matrix index associated with that base, and returns the smallest distance as the variable XTNK. XTNK is then used by DYNAM to compute the maximum permissible fuel offload per tanker at each stage, $FTNK_i$.

DISBAS is also used to compute the distance between each stage and all diversion bases. It also selects the base with the smallest distance, stores the matrix index for that base, and returns that distance as the variable XDIV. XDIV is then used by CONST and SCONST to calculate the diversion fuel required at each stage.

Subroutine DISXRT

Subroutine DISXRT uses the same relationships presented above to compute the distance in nautical miles between each point on the airlifter's route of flight. DISXRT returns the distance as the variable XRT_i.

Appendix C

Computer Code

This appendix presents the computer code for the AR optimization model and describes the variables used in the program. The general structure of DYNAM and its subroutines were obtained from Reference 16. DYNAM has the built-in capability to handle more than one decision variable per stage. It also has three options for types of problems to be solved and program output, all of which are described in Appendix B and are selectable by the user. These options were left in the program to allow the user some flexibility in applying the model operationally.

DYNAM is written in ANSI Fortran V. For the 20 stage model, the program requires 65,000 words of core memory on the Control Data Corporation (CDC) 6600 computer. Execution time was between 120 and 140 seconds for the 12 stage scenario used in the analysis of Chapter IV.

<u>Description</u> <u>of</u> <u>Variables</u>

ALPHA - Reflection factor (see Appendix D).

BETA - Convergence parameter (COMPLX).

DECIS - Decision variable.

DECMAX - Optimal value of decision variable for each stage.

DELTA - Explicit constraint violation correction.

DIVLA - Latitude of diversion base.

DIVLO - Longitude of diversion base.

F - Value of objective function (FUNC).

FMAX - Optimal return for the entire system.

FTNK - Maximum fuel offload capability of one tanker at each stage.

G - Lower decision value constraint (CONST).

GAMMA - Convergence parameter (COMPLX).

H - Upper decision value constraint (CONST).

I - Complex point index (COMPLX).

IBAC - Stage index during backward stage recursion (I-BACkwards).

IEV1 - Index of point with lowest function value (COMPLX).

IEV2 - Index of point with highest function value (COMPLX).

IOPT - Code for sign of function at optimum. IOPT = -1 for minimum.

IPRINT - Code to select printing of intermediate results.

IPROB - Code for defining type of problem.

IT - Iteration number (COMPLX).

ITMAX - Maximum number of iterations for stage optimization (COMPLX).

K - Number of points in the Complex.

K1 - Do loop limit (COMPLX).

KODE - Key used to:

1. Determine if implicit constraints are provided (COMPLX).

2. Determine if high or low constraint is desired (DYNAM).

3. Determine if system output is desired (DYNAM).

LOAD	-	Airlifter cargo load.
M	-	Number of constraints on decision variables at each stage.
MACLA	-	Latitude of points along airlifter route of flight.
MACLO	-	Longitude of points along airlifter route of flight.
NALFTR	-	Type of airlifter.
NBAS	-	Number of possible tanker departure bases.
NDECIS	-	Number of decision variables at each stage.
NDIV	-	Number of possible diversion bases.
NI	-	Input unit number.
NO	-	Output unit number.
NSTAGE	-	Number of stages, or points along the airlifter's route of flight.
NSTEP	-	Number of intervals that the range of state variables is divided.
NTNKR	-	Type of tanker.
OPWT	-	Operating weight of the airlifter.
OPWTT	-	Operating weight of the tanker.
R	-	Random number between 0 and 1.
RET	-	Value of return variable during optimization.
RETMAX	-	Optimal value of return function for each stage.
S	-	Value of state variable during optimization.
SCON	-	Value of state variable limit (SCONST).
SHIGH	-	Upper bound on state value.
SLOW	-	Lower bound on state value.
SMAX	-	Optimum state variable values.
SN	-	System input state.
SO	-	System output state.

TAS	-	Airlifter true airspeed.
TAST	-	Tanker true airspeed.
TNKLA	-	Latitude of tanker departure base.
TNKLO	-	Longitude of tanker departure base.
WF	-	Average wind factor between stages.
X	-	Value of decision variable during optimization.
XC	-	Centroid (COMPLX--see Appendix D).
XDIV	-	Great circle distance from stage to closest diversion base (nautical miles).
XTNK	-	Great circle distance from stage to closest tanker base (nautical miles).
XTNKM	-	Index of closest tanker base from each stage.

```
PROGRAM DYNAM (INPUT,OUTPUT,TAPE50=INPUT,TAPE60=OUTPUT)

MAIN LINE PROGRAM FOR DYNAMIC PROGRAMMING ALGORITHM

THIS PROGRAM DETERMINES THE OPTIMAL AERIAL REFUELING
POINTS AND FUEL OFFLOAD AT EACH POINT TO MINIMIZE THE
FUEL CONSUMED BY BOTH AIRLIFTER AND TANKER.  IT ALSO
DETERMINES THE OPTIMAL TANKER BASE FOR EACH REFUELING.
INPUT FORMAT IS FREEFIELD.  THE PROGRAM REQUIRES THE
FOLLOWING DATA ARRANGED BY CARD AS FOLLOWS


CARD #1 - NUMBER OF POINTS DEFINING AIRLIFTER ROUTE
          OF FLIGHT.

      2 - COORDINATES (DEGREES.DECIMAL) OF EACH POINT,
          BEGINNING WITH LEVEL OFF POINT AND ENDING
          WITH DESCENT POINT.

      3 - NUMBER OF TANKER BASES TO BE CONSIDERED (10 MAX).

      4 - COORDINATES OF TANKER BASES.

      5 - NUMBER OF AVAILABLE AIRLIFTER DIVERSION BASES (10 MAX).

      6 - COORDINATES OF DIVERSION BASES.  THIS SHOULD INCLUDE
          DEPARTURE AND DESTINATION IF THEY QUALIFY AS
          DIVERSION BASES.

      7 - WIND FACTOR FOR EACH LEG.

      8 - TAKEOFF FUEL, REQUIRED OVERHEAD DESTINATION
          FUEL, AIRLIFTER CARGO LOAD (ALL IN POUNDS).

      9 - TYPE OF AIRLIFTER
          1 - C-5A
          2 - C-141B

     10 - TYPE OF TANKER
          1 - KC-135A
          2 - KC-10A

THIS PROGRAM WILL HANDLE INITIAL VALUE (GIVEN TAKEOFF
FUEL), FINAL VALUE (GIVEN REQUIRED OVERHEAD DESTINATION
FUEL), OR BOTH INITIAL VALUE/FINAL VALUE PROBLEMS.
```

```
      DIMENSION X(20,1), F(20), DECMAX(20,1), NTOT(20), SMAX(20),
     1R(3,1), G(3), H(3), XC(2), RETMAX(20), DUMB(20), DUMB1(20),
     2TNKLA(10), TNKLO(10), DIVLA(10), DIVLO(10), XTNK(20)
      COMMON N, S(20,1 1), IP, RET(20,101), SO, DECIS(20,1,101),
     1FOR(20,1 1), NTOTN, NTOTM1, IOPT, IAS, OPWT, LOAD, XDIV(20),
     2FTNK(20), XFT(20), NSTEP, NSTAGE, SLOW, WF(20), XMAXFA,
     3XMAXFT, XMAXGW
      INTEGER GAMMA, XTNKM(20), XDIVM(20)
      REAL LOAD, MACLA(20), MACLO(20)

      NI = 5
      NO = 6

      OPTIMIZATION PARAMETERS AND INITIALIZATION.  THESE PARAMETERS
      ARE THE ONLY ONES CHANGEABLE BY THE USER

      IPRINT:   0 - ONLY PRINTS FINAL RESULTS
                1 - ALL SEARCH VALUES AND TABLES ARE PRINTED
                2 - ONLY VALUES AND TABLES ARE PRINTED AFTER
                    EACH STAGE IS OPTIMIZED

      IPROB:    1 - INITIAL VALUE PROBLEM
                2 - FINAL VALUE PROBLEM
                3 - INITIAL VALUE/FINAL VALUE PROBLEM

      ITMAX = 150
      IPRINT = 0
      IPROB = 3
      IOPT = -1
      ALPHA = 1.3
      BETA = 0.01
      GAMMA = 5
      DELTA = 1.0E-06
      FMAX = 0.0
      N = 1
      M = 1
      NDECIS = 1
      CALL PANSET (13)

      DATA INPUT

      READ *,NSTAGE
      DO 116 I=1,NSTAGE
      READ *, MACLA(I), MACLO(I)
      PRINT *, MACLA(I), MACLO(I)
  116 CONTINUE
      READ *,NBASE
      DO 118 I=1,NBASE
      READ *, TNKLA(I), TNKLO(I)
      PRINT *, TNKLA(I), TNKLO(I)
  118 CONTINUE
```

```
      READ *,NDIV
      DO 124 I=1,NDIV
      READ *, DIVLA(I), DIVLO(I)
      PRINT *, DIVLA(I), DIVLO(I)
  124 CONTINUE
       DO [illegible]1 I=1,NSTAGE-1
      READ *,WF(I)
      PRINT *,WF(I)
  [illegible]1 CONTINUE
      READ *,SN, SO, LOAD
      PRINT *, SN, SO, LOAD
      READ *,NTNKR, NALFTR
      PRINT *,NTNKR, NALFTR

      SO2 = SO
      IPHOLD = IPRINT
      WRITE (NO,[illegible]15)
  [illegible]15 FORMAT (1H1,1[illegible]X,23HDYNAMIC PROGRAMMING PROCEDURE )

  2[illegible] WRITE (NO,[illegible]23) NSTAGE
  [illegible]23 FORMAT (//,2X,16HOPTIMIZATION OF ,I2,39H STAGE INITIAL AND FINAL V
     1ALUE PROBLEM   )

      INITIALIZE AIRCRAFT PARAMETERS

      IF (NALFTR.EQ.1) THEN
      TAS = 45[illegible]
      OPWT = 354[illegible]
      XMAXFA = 3181[illegible]
      XMAXGW = 712[illegible]
      ELSE
      TAS = 432.[illegible]
      OPWT = 1[illegible]88[illegible]
      XMAXFA = 153352
      XMAXGW = 3231[illegible]
      ENDIF
      IF (NTNKR.EQ.1) THEN
      TAST = 479.[illegible]
      XMAXFT = 163[illegible]
      OPWTT = 1[illegible]
      ELSE
      TAST = 479.[illegible]
      XMAXFT = 345[illegible]
      OPWTT = 248473
      ENDIF

      USE SUBROUTINES TO CALCULATE DISTANCES FROM COORDINATES

      CALL DTSXRT (MACLA,MACLO,XFT,NSTAGE)
      CALL DISBAS (MACLA,MACLO,TNKLA,TNKLO,NSTAGE,XTNK,NBASE,XTNKM)
      CALL DISBAS (MACLA,MACLO,DIVLA,DIVLO,NSTAGE,XDIV,NDIV,XDIVM)
```

```
      THIS ROUTINE REVERSES INDEX ORDER FOR RECURSIVE ANALYSIS
      (STARTING AT STAGE 1 - DESTINATION)

      DO 70 I=1,NSTAGE-1
      DUMB(I) = XFT(I)
      DUMB1(I) = WF(I)
   70 CONTINUE
      DO 71 I=1,NSTAGE-1
      XFT(I) = DUMB(NSTAGE-I)
      WF(I) = DUMB1(NSTAGE-I)
   71 CONTINUE
      DO 72 I=1,NSTAGE
      DUMB(I) = XTNK(I)
      DUMB1(I) = XDIV(I)
   72 CONTINUE
      DO 73 I=1,NSTAGE
      XTNK(I) = DUMB(NSTAGE+1-I)
      XDIV(I) = DUMB1(NSTAGE+1-I)
   73 CONTINUE

      CALCULATES AVAILABLE FUEL FOR OFFLOAD GIVEN TANKER DEPARTURE
      BASE.  THIS IS DONE FOR EACH POINT ON THE AIRLIFTER ROUTE
      OF FLIGHT.

      IF (NTNKR.EQ.1) THEN
      DO 74 I=1,NSTAGE
      PRINT *,'XTNK',I,' = ',XTNK(I)
      FTNK(I) = XMAXFT - ((2.54E-6*TAST**2.24*(XMAXFT/2.+OPWTT)**.916)
     1*(XTNK(I)/TAST)*6.5**2) - 25063
   74 CONTINUE
      ELSE
      DO 76 I=1,NSTAGE
      FTNK(I) = XMAXFT-((7.995E-6*TAST**1.1214*(XMAXFT/2.+OPWTT)
     1 **.3795)*(XTNK(I)/TAST)*6.5*2)-42722
   76 CONTINUE
      ENDIF

      25063 LBS INCLUDES
         11263 LBS - 1 HOUR ORBIT
         7300 LBS - ADDITIONAL FUEL FOR CLIMB, DESCENT, APPPOACH
                    AND LANDING.
         6500 LBS - 45 MINUTES HOLDING AT DESTINATION.


      24722 LBS INCLUDES
         17322 LBS - 1 HOUR ORBIT
         13900 LBS - ADDITIONAL FUEL FOR CLIMB, DESCENT, APPROACH
                     AND LANDING.
         11500 LBS - 45 MINUTES HOLDING AT DESTINATION
```

```
C     RECURSIVE ANALYSIS OF STAGES STARTING AT STAGE 1

  10  NS = N
      DECIS(N,1,1) = 1.0
      K = 3*NDECIS
      NDEC = NDECIS

      IF (N.EQ.NSTAGE) THEN
      SLOW = SN
      GO TO 75
      ENDIF

      KODE = 0
      CALL SCONST (N,SLOW,KODE)
  75  KODE = 1
      CALL SCONST (N,SHIGH,KODE)
      PRINT *,'SHIGH = ',SHIGH
      PRINT *,'SLOW = ',SLOW

      PRINT *,'XRT',N,' = ',XRT(N)
      PRINT *,'XDIV',N,' = ',XDIV(N)
      PRINT *,'FINK',N,' = ',FINK(N)

      NSTEP = 10
      STEP = (SHIGH-SLOW)/NSTEP
 215  NTOT(N) = NSTEP + 1
      NTOTN = NTOT(N)
      IF (N.GE.2) NTOTM1 = NTOT(N-1)

C     PERFORM SEARCH AT STAGE N FOR GIVEN STATE VALUE

      DO 100 IP=1,NTOTN

      IF (IPRINT - 1) 28, 28, 26
  26  IPRINT = 1
  28  IM1 = IP - 1
      S(N,IP) = SLOW + IM1*STEP

      DO 30 J=1,NDEC
  30  X(1,J) = DECIS(N,J,1)

      CALL CONPLX(NDEC,M,K,ITMAX,ALPHA,BETA,GAMMA,DELTA,X,F,IT,IEV2,NO,
     1 IPRINT,R,G,H,XC)

      FOP(N,IP) = IOPT * F(IEV2)
      DO 40 J=1,NDEC
  40  DECIS(N,J,IP) = X(IEV2,J)
      IF (IPHOLD - 1) 100, 45, 45
  45  IPRINT = IPHOLD
  50  IF (IP - NTOTN) 100,55,55
  55  WRITE (NO,103) N
 103  FORMAT (//,10X,20HTABLE OF VALUES FOR STAGE ,I2)
```

```
      WRITE (NO, 115)
  115 FORMAT (/,2X,11HSTATE VALUE,7X,14HFUNCTION VALUE,11X,
     1 15HDECISION VALUES ,/)
      DO 60 II = 1,NTOTN
   60 WRITE (NO, 64) S(N,II), FOP(N,II), (DECIS(N,J,II), J=1,NDEC)
   64 FORMAT (2X,1PE14.6,4X,1PE14.6,3(4X,1PE14.6))

  100 CONTINUE

      IF (N - NSTAGE) 110, 120, 120
  110 N = N + 1
      GO TO 10

      SOLVING FORWARDS FOR OPTIMUM PATH

  120 DO 200 II=1,NSTAGE
      IBAC = NSTAGE + 1 - II
      NTOTN = NTOT(IBAC)
      NDEC = NDECIS
      SMAX(IBAC) = S(IBAC,NTOTN)
      RETMAX(IBAC) = RET(IBAC,NTOTN)
      DO 128 K=1,NDEC
  128 DECMAX(IBAC,K) = DECIS(IBAC,K,NTOTN)
      IF (IBAC.NE.NSTAGE) GO TO 160
      FMAX = FOP(IBAC,NTOTN)
      DO 150 J=1,NTOTN
      GO TO (122,124,122), IPROB
  122 IF (S(IBAC,J) - SN) 150, 130, 130
  124 IF (FOP(IBAC,J) - FMAX) 150, 130, 130
  130 FMAX = FOP(IBAC,J)
      DO 140 K=1,NDEC
  140 DECMAX(IBAC,K) = DECIS(IBAC,K,J)
      SMAX(IBAC) = S(IBAC,J)
      RETMAX(IBAC) = RET(IBAC,J)
      IF (IPROB.NE.2) GO TO 200
  150 CONTINUE
      GO TO 200
  160 KODE = 0
      CALL TRANS (IBAC,SMAX,DECMAX,SO,KODE)
      DO 190 J=1,NTOTN
      IF (S(IBAC,J) - SMAX(IBAC)) 190,170,170
  170 SMAX(IBAC) = S(IBAC,J)
      RETMAX(IBAC) = RET(IBAC,J)
      DO 180 K=1,NDEC
  180 DECMAX(IBAC,K) = DECIS(IBAC,K,J)
      GO TO 200
  190 CONTINUE
  200 CONTINUE
```

```
      KODE = 1
      CALL TRANS (IBAC,SMAX,DECMAX,SO,KODE)
      DO 300 N=1,NSTAGE

      NDEC = NDECIS
      DO 210 II=1,NDEC
  210 X(N,II) = DECMAX (N,II)
      IP = 1
      CALL RETURS (X,N)
  300 CONTINUE

      WRITE (NO, 511) FMAX
  511 FORMAT (///,2X,24HSYSTEM OPTIMUM RETURN = ,1PE16.8)
      WRITE (NO, 512)
  512 FORMAT (//,10X,21HMAXIMUM STAGE RETURNS )
      DO 400 I=1,NSTAGE
  400 WRITE (NO,513) I, RETMAX(I)
  513 FORMAT (/,2X,6HSTAGE ,I2,1 H RETURN = ,1PE16.8)
      WRITE (NO,514)
  514 FORMAT (//,10X,17HOPTIMUM DECISIONS )
      DO 500 J=1,NSTAGE
      NDEC = NDECIS
      WRITE (NO,516) J, (J, I, X(J,I), I=1,NDEC)
  516 FORMAT (/,2X,6HSTAGE ,I2,6X,2(4X,2HX(,I1,1H,,I1,4H) = ,1PE16.8))
  500 CONTINUE
      WRITE (NO,518) SO
  518 FORMAT (/,2X,26HTHE SYSTEM OUTPUT STATE = ,1PE16.8)
      NSTM1 = NSTAGE - 1
      DO 600 JJ=1,NSTM1
      WRITE (NO, 519) JJ, SMAX(JJ)
  519 FORMAT (/,2X,25HTHE INPUT STATE TO STAGE ,I2,3H = ,1PE16.8)
  600 CONTINUE
      SN = SMAX(NSTAGE)
      WRITE (NO, 517) SN
  517 FORMAT ( /,2X,25HTHE SYSTEM INPUT STATE = ,1PE16.8)
      WRITE (NO,501)
  501 FORMAT (////,10X,38HAERIAL REFUELING OPTIMIZATION PROGRAM )
      WRITE (NO,502)
  502 FORMAT (/,20X,19HSUMMARY OF RESULTS )
      WRITE (NO,503)
  503 FORMAT (/////,16X,27HAIRLIFTER ROUTE OF FLIGHT: )
      RAD = 57.29577951
      DO 504 I=1,NSTAGE
      MACLA(I) = MACLA(I)*RAD
      MACLO(I) = MACLO(I)*RAD
  504 CONTINUE
      DO 505 I=1,NBASE
      TNKLA(I) = TNKLA(I)*RAD
      TNKLO(I) = TNKLO(I)*RAD
  505 CONTINUE
```

```
      DO 5 6 I=1,NDIV
      DIVLA(I) = DIVLA(I)*RAD
      DIVLO(I) = DIVLO(I)*RAD
 516  CONTINUE
      WRITE (NO,507)
 507  FORMAT (///,1 X,5HSTAGE,5X,11HCOORDINATES,6X,11HWIND FACTOR,4X,
     1 1 HDISTANCE (NM) ,/)
      XRT(NSTAGE) =
      WF(NSTAGE) =
      DO 5 8 I=1,NSTAGE
      IBAC = NSTAGE +1-I
      WRITE (NO,5 9) I,MACLA(IBAC),MACLO(IBAC),WF(I),XRT(I)
 509  FORMAT (/,12X,I2,3X,F6.2,3X,F7.2,5X,F5.3,1 X,F7.2)
 5 8  CONTINUE
      WRITE (NO,51 )
 51   FORMAT (////,1 X,24HTANKER DEPARTURE BASES: )
      WRITE (NO,511)
 511  FORMAT (///,17X,8HBASE NO.,4X,12HCOORDINATES ,/)
      DO 512 I=1,NBASE
      WRITE (NO,513) I,TNKLA(I),TNKLO(I)
 513  FORMAT (/,19X,I2,5X,F6.2,3X,F7.2)
 512  CONTINUE
      WRITE (NO,514)
 514  FORMAT (////,16X,26HAIRLIFTER DIVERSION BASES )
      WRITE (NO,515)
 515  FORMAT (///,16X,8HBASE NO.,6X,12HCOORDINATES ,/)
      DO 516 I=1,NDIV
      WRITE (NO,517) I,DIVLA(I),DIVLO(I)
 517  FORMAT (/,18X,I2,8X,F6.2,3X,F7.2)
 516  CONTINUE
      IF (NALFTR.EQ.1) THEN
      WRITE (NO,518)
 518  FORMAT (////,16X,16HTYPE AIRLIFTER: ,16X,5HC-5A )
      ELSE
      WRITE (NO,519)
 519  FORMAT (////,16X,16HTYPE AIRLIFTER: ,16X,7HC-141B )
      ENDIF
      WRITE (NO,520) LOAD
 52   FORMAT (/,16X,12HCARGO LOAD: ,2 X,F7. ,2X,5HLBS. )
      WRITE (NO,521) SN
 521  FORMAT (/,16X,14HTAKEOFF FUEL: ,18X,F7. ,2X,5HLBS. )
      WRITE (NO,522) SO2
 522  FORMAT (/,16X,32HREQD OVERHEAD DESTINATION FUEL: ,F7. ,2X,5HLBS. )
      IF (NTNKR.EQ.1) THEN
      WRITE (NO,523)
 523  FORMAT (////,16X,13HTYPE TANKER: ,19X,6HKC-135A )
      ELSE
```

```
      WRITE (NO,524)
  524 FORMAT (////,16X,13HTYPE TANKER: ,19X,7HKC-10A )
      ENDIF
      WRITE (NO,525)
  525 FORMAT (/////,14X,3HNO.,5X,7HOFFLOAD,5X,10HTOTAL FUEL,5X,
     1 9HDIVERSION,3X,6HTANKER,4X,5HFUEL )
      WRITE (NO,526)
  526 FORMAT (/,4X,5HSTAGE,3X,7HTANKERS,4X,5H(LBS),4X,14HCONSUMED (LBS),
     1 5X,4HBASE,7X,4HBASE,5X,6HSTATE ,//)
      DO 527 I=1,NSTAGE
      IBAC = NSTAGE+1-I
      FLOAD = X(1,1)*FTNK(I)
      WRITE (NO,528) I,X(I,1),FLOAD,FETMAX(I),XDIVM(IBAC),XTNKM(IBAC),
     1 SMAX(I)
  528 FORMAT (/,6X,I2,5X,F4.2,5X,F7.0,7X,F7.0,9X,I2,9X,I2,5X,F7.0)
  527 CONTINUE
      WRITE (NO,529) FMAX
  529 FORMAT (/////,15X,29HOPTIMUM TOTAL FUEL CONSUMED: ,3X,F8.0,
     1 3X,5HLBS. )

      END
```

```
      SUBROUTINE COMPLX (N,M,K,ITMAX,ALPHA,BETA,GAMMA,DELTA,X,F,IT,IEV2,
     1 NO,IPRINT,R,G,H,XC)
C     COORDINATES SPECIAL PURPOSE SUBROUTINES
C
C     ARGUMENT LIST
C
C     IT     = ITERATION INDEX
C     IEV1   = INDEX OF POINT WITH MINIMUM FUNCTION VALUE
C     IEV2   = INDEX OF POINT WITH MAXIMUM FUNCTION VALUE
C     I      = POINT INDEX
C     KODE   = CONTROL KEY USED TO DETERMINE IF IMPLICIT CONSTRAINTS
C              ARE PROVIDED
C     K1       DO LOOP LIMIT
C
C     ALL OTHERS PREVIOUSLY DEFINED IN MAIN LINE
C
      DIMENSION X(K,M), R(K,N), F(K), G(M), H(M), XC(N)
      INTEGER GAMMA
C
      IT = 1
      KODE = 0
      IF (M-N) 20,20,10
   10 KODE = 1
   20 CONTINUE
      DO 40  II=2,K
      DO 30  J=1,N
   30 X(II,J) = 0.0
   40 CONTINUE
C
      DO 45 II=2,K
      DO 45 JJ=1,N
C
C     GENERATES RANDOM NUMBERS FOR CALCULATING COMPLEX
C
      R(II,JJ) = RANF()
C
   45 CONTINUE
C
      IF (IPRINT) 46, 43, 46
   46 WRITE (NO,111)
  111 FORMAT (//,2X,10HPARAMETERS )
      WRITE (NO,112) N, M, K, ITMAX, IC, ALPHA, BETA, GAMMA, DELTA
  112 FORMAT (//,2X,4HN = ,I2,3X,4HM = ,I2,3X,4HK = ,I2,2X,8HITMAX = ,
     1I4,2X,5HIC = ,I2,//2X,8HALPHA = ,F5.2,5X,7HBETA = ,F10.5,3X,
     28HGAMMA = ,I2,3X,8HDELTA = ,E12.6)
      WRITE (NO,113)
  113 FORMAT (//,2X,14HRANDOM NUMBERS)
      DO 47  J=2,K
      WRITE (NO,114) (J, I, R(J,I), I=1,N)
  114 FORMAT (/,2X,3(2HR(,I2,1H,,I2,4H) = ,F6.4,2X))
   47 CONTINUE
```

```
C     CALCULATE COMPLEX POINTS AND CHECK AGAINST CONSTRAINTS
C
   48 DO 65 II=2,K
      DO 5  J=1,N
      I = II
      CALL CONST (N,M,K,X,G,H,I)
      X(II,J) = G(J) + R(II,J)*(H(J)-G(J))
    5 CONTINUE
      K1 = II
      CALL CHECK (N,M,K,X,G,H,I,KODE,XC,DELTA,K1)
      IF (II-2) 51, 51, 55
   51 IF (IPRINT) 52, 65, 52
   52 WRITE (NO, 18)
   18 FORMAT (//,2X,30HCOORDINATES OF INITIAL COMPLEX)
      IO = 1
      WRITE (NO, 19) (IO, J, X(IO,J), J=1,N)
   19 FORMAT (/,3(2X,2HX(,I2,1H,,I2,4H) = ,1PE13.6))
   55 IF (IPRINT) 56, 65, 56
   56 WRITE (NO, 19) (II, J, X(II,J), J=1,N)
   65 CONTINUE
      K1 = K
      DO 7  I=1,K
      CALL FUNC (N,M,K,X,F,I)
    7 CONTINUE
      KOUNT = 1
      IA = 0
C
C     FIND POINT WITH LOWEST FUNCTION VALUE
C
      IF (IPRINT) 72, 80, 72
   72 WRITE (NO, 21)
   21 FORMAT (/,2X,22HVALUES OF THE FUNCTION )
      WRITE (NO, 22) (J, F(J), J=1,K)
   22 FORMAT (/,3(2X,2HF(,I2,4H) = ,1PE13.6))
   80 IEV1 = 1
      DO 10 ICM=2,K
      IF (F(IEV1)-F(ICM)) 10, 10, 9
    9 IEV1 = ICM
   10 CONTINUE
C
C     FIND POINT WITH HIGHEST FUNCTION VALUE
C
      IEV2 = 1
      DO 12 ICM=2,K
      IF (F(IEV2)-F(ICM)) 11,11,12
   11 IEV2 = ICM
   12 CONTINUE
```

```
      CHECK CONVERGENCE CRITERIA

      IF (F(IEV2) - F(IEV1) - (ABS(BETA*F(IEV1)))) 14 , 130, 13
13    KOUNT = 1
      GO TO 15
14    KOUNT = KOUNT + 1
      IF (KOUNT-GAMMA) 15 ,24 ,24

      REPLACE POINT WITH LOWEST FUNCTION VALUE

15    CALL CENTR (N,M,K,IEV1,I,XC,X,K1)
      DO 16  JJ=1,N
16    X(IEV1,JJ) = (1.  +ALPHA)*(XC(JJ))-ALPHA*(X(IEV1,JJ))
      I = IEV1
      CALL CHECK (N,M,K,X,G,H,I,KODE,XC,DELTA,K1)
      CALL FUNC (N,M,K,X,F,I)

      REPLACE NEW POINT IF IT REPEATS AS LOWEST FUNCTION VALUE

17    IEV2 = 1
      DO 19  ICM=2,K
      IF (F(IEV2)-F(ICM)) 19 ,19 ,18
18    IEV2 = ICM
19    CONTINUE
      IF (IEV2-IEV1) 22 ,2  ,22
2     DO 21  JJ=1,N
      X(IEV1,JJ)=(X(IEV1,JJ) + XC(JJ))/2.
21    CONTINUE
      I = IEV1
      CALL CHECK (N,M,K,X,G,H,I,KODE,XC,DELTA,K1)
      CALL FUNC (N,M,K,X,F,I)
22    CONTINUE
      IF (IPRINT) 23 , 228, 23
23    WRITE (NO, 23) IT
 23   FORMAT (//2X,17HITERATION NUMBER ,I5)
      WRITE (NO, 24)
 24   FORMAT (/,2X,3 HCOORDINATES OF CORRECTED POINT)
      WRITE (NO, 19) (IEV1, JC, X(IEV1,JC), JC=1,N)
      WRITE (NO, 21)
      WRITE (NO, 22) (I, F(I), I=1,K)
      WRITE (NO, 25)
 25   FORMAT (/,2X,27HCOORDINATES OF THE CENTROID)
      WRITE (NO, 26) (JC, XC(JC), JC=1,N)
 26   FORMAT (/,2X,3(2HX(,I2,5H,C) = ,1PE14.6,4X))
228   IT = IT + 1
      IF (IT-ITMAX) ,3 ,24
24    RETURN
      END
```

```
      SUBROUTINE CENTR (N,M,K,IEV1,I,XC,X,K1)

      DIMENSION X(K,M), XC(N)

      DO 20 J=1,N
      XC(J) = 0.0
      DO 10 IL=1,K1
   10 XC(J) = XC(J) + X(IL,J)
      RK = K1
   20 XC(J) = (XC(J)-X(IEV1,J))/(RK-1.0)
      RETURN
      END

      SUBROUTINE CHECK (N,M,K,X,G,H,I,KODE,XC,DELTA,K1)

      ARGUMENT LIST

      ALL ARGUMENTS DEFINED IN MAIN LINE AND CONSX

      DIMENSION X(K,M), G(M), H(M), XC(N)

   10 KT = 0
      CALL CONST (N,M,K,X,G,H,I)

      CHECK AGAINST EXPLICIT CONSTRAINTS

      DO 50 J=1,N
      IF (X(I,J)-G(J)) 20,20,30
   20 X(I,J) = G(J) + DELTA
      GO TO 50
   30 IF (H(J)-X(I,J)) 40,40,50
   40 X(I,J) = H(J) - DELTA

   50 CONTINUE
      IF (KODE) 110,110,60

      CHECK AGAINST THE IMPLICIT CONSTRAINTS

   60 NN = N + 1
      DO 100 J=NN,M
      CALL CONST (N,M,K,X,G,H,I)
      IF (X(I,J)-G(J)) 80,70,70
   70 IF (H(J)-X(I,J)) 80,100,100
   80 IEV1 = I
      KT = 1
      CALL CENTR (N,M,K,IEV1,I,XC,X,K1)
      DO 90 JJ=1,N
      X(I,JJ) = (X(I,JJ) + XC(JJ))/2.0
   90 CONTINUE
  100 CONTINUE
      IF (KT) 110, 110, 10
  110 RETURN
      END
```

```
      SUBROUTINE FUNC (N,M,K,X,F,I)

      DIMENSION X(K,M), F(K)
      DIMENSION SFUNC(20), XFUNC(20,10)

      COMMON NUMST, S(20,101), IP, RET(20,101), SO, DECIS(20,1,101),
     1FDP(20,101), NTOTN, NTOTM1, IOPT, IAS, OPWT, LOAD, XDIV(20),
     2FTNK(20), XNT(20), NSTEP, NSTAGE, SLOW, WF(20), XMAXFA, XMAXFT
      REAL LOAD

      CALL RETURS(X,I)

      IF (NUMST - 1) 1 , 10, 20

   10 F(I) = RET(NUMST,IP)
      GO TO 99

   20 NUMT = NUMST - 1
      KODE = 0
      FSTM1 = FDP(NUMT,NTOTM1)
      SFUNC(NUMST) = S(NUMST,IP)
      DO 30  JF=1,N
   30 XFUNC(NUMST,JF) = X(I,JF)
      CALL TRANS (NUMT,SFUNC,XFUNC,SO,KODE)
      IF (NUMT.EQ.1.AND.S(NUMT,1).GT.SFUNC(NUMT)) THEN
      FSTM1 = 1.0E+6
      GO TO 60
      ENDIF
      DO 50  IS=1,NTOTM1
      IF (S(NUMT,IS) - SFUNC(NUMT)) 50, 40, 40
   40 FSTM1 = FDP(NUMT,IS)
      GO TO 60
   50 CONTINUE

   60 F(I) = RET(NUMST,IP)  + FSTM1

   99 F(I) = IOPT * F(I)
      RETURN
      END
```

```
      SUBROUTINE SCONST (N,SCON,KODE)

      COMMON NUMST, S(20,1,11), IP, RET(20,1,11), SO, DECIS(20,1,11),
     1FDP(20,1,11), NTOTN, NTOTM1, IOPT, TAS, OPWT, LOAD, XDIV(20),
     2FTNK(20), XRT(20), NSTEP, NSTAGE, SLOW, WF(20), XMAXFA,
     3XMAXFT, XMAXGW
      REAL LOAD

      IF (KODE) 100, 100, 200

  100 IF (N.EQ.1) THEN
      SCON = SO
      GO TO 99
      ENDIF
      SCON1 = (1.81E-3*TAS**.5563*(XMAXFA/2.+LOAD+OPWT)**.8345*
     1 XDIV(N)/(TAS+WF(N))))*6.5
      SCON2 = (1.81E-3*TAS**.5563*(XMAXFA/2.+LOAD+OPWT)**.8345*
     1 XRT(N-1)/(TAS+WF(N))))*5.5
      IF (SCON2.LT.SCON1) THEN
      SCON = SCON2
      ELSE
      SCON = SCON1
      ENDIF
      GO TO 99
  200 SCON = XMAXFA
      IF ((XMAXFA+OPWT+LOAD).GT.XMAXGW) THEN
      SCON = XMAXGW-OPWT-LOAD
      ENDIF

      IF (N.EQ.1) THEN
      SCON = SO
      ENDIF


   99 RETURN
      END
```

```
      SUBROUTINE CONST (N,M,K,X,G,H,I)

      DIMENSION X(K,M), G(M), H(M)

      COMMON NUMST, S(20,101), IP, RET(20,101), SO, DECIS(20,1,101),
     1FDF(20,101), NTOTN, NTOTM1, IOPT, TAS, OPWT, LOAD, XDIV(20),
     2FTNK(20), XFT(20), NSTEP, NSTAGE, SLOW, WF(20), XMAXFA,
     3XMAXFT, XMAXGW
      REAL LOAD

      IF(FTNK(NUMST).LT.0) THEN
      H(1) = 0
      G(1) = 0
      RETURN
      ENDIF
      IF (NUMST.EQ.1) THEN
      G(1) = 0.0
      GO TO 99
      ENDIF
      F1 = (1.81E-3*TAS**.5593*(OPWT+LOAD)+XMAXFA/2.)**.8345
     1 *XFT(NUMST-1)/(TAS+WF(NUMST-1)))*5.5
      F2 = (1.81E-3*TAS**.5593*(OPWT+LOAD)+XMAXFA/2.)**.8345
     1 *XDIV(NUMST-1)/(TAS+WF(NUMST-1))) 5.5
      IF ((F1+F2).GT.S(NUMST,IP)) THEN
      G(1) = (F1+F2-S(NUMST,IP))/FTNK(NUMST)
      ELSE
      G(1) = 0.0
      ENDIF
   99 H(1) = (XMAXFA-SLOW)/FTNK(NUMST)
      IF ((XMAXFA+OPWT+LOAD).GT.XMAXGW) THEN
      H(1) = ((XMAXGW-OPWT-LOAD)-SLOW)/FTNK(NUMST)
      ENDIF

      RETURN
      END
```

```
      SUBROUTINE TRANS (IBAC,SMAX,DECMAX,SO,KODE)

      DIMENSION SMAX(20), DECMAX(20,1)
      COMMON N, S(20,101), IP, RET(20,101), SO1, DECIS(20,1,101),
     1FDP(20,101), NTOTN, NTOTM1, IOPT, TAS, OPWT, LOAD, XDIV(20),
     2FTNK(20), XFT(20), NSTEP, NSTAGE, SLOW, WF(20)
      REAL LOAD

      IF (KODE) 10, 10, 20

   10 IBACP1 = IBAC + 1
      SMAX(IBAC) = SMAX(IBACP1)-((1.81E-3*TAS**.5503*(DECMAX(IBACP1,1)
     1 *FTNK(IBACP1)+SMAX(IBACP1)+OPWT+LOAD)**.8345)*XRT(IBAC)/
     2 (TAS+WF(IBAC)))*5.50+DECMAX(IBACP1,1)*FTNK(IBACP1)
   20 SO = SMAX(1)

   99 RETURN
      END




      SUBROUTINE RETURS (X,I)

      DIMENSION X(20,1)

      COMMON N, S(20,101), IP, RET(20,101), SO, DECIS(20,1,101),
     1FDP(20,101), NTOTN, NTOTM1, IOPT, TAS, OPWT, LOAD, XDIV(20),
     2FTNK(20), XFT(20), NSTEP, NSTAGE, SLOW, WF(20), XMAXFA, XMAXFT
      REAL LOAD

      IF (N.NE.1) THEN
      RET(N,IP) = (INT(X(I,1)+.9999)*(XMAXFT-FTNK(N)))+((1.81E-3*TAS
     1 **.5503*(OPWT+LOAD+S(N,IP)+X(I,1)*FTNK(N))**.8345)*XRT(N-1)/
     2 (TAS+WF(N-1)))*5.5
      ELSE
      RET(N,IP) = INT(X(I,1)+.9999)*(XMAXFT-FTNK(N))
      ENDIF
      RETURN
      END
```

```
      SUBROUTINE DISBAS(MACLA,MACLO,XP1,XP2,NSTAGE,X2,NBAS,X3)

      COMPUTES GREAT CIRCLE DISTANCE - TANKER AND DIVERSION BASES

      DIMENSION XP1(10), XP2(10), X1(20,10), X2(20)
      INTEGER X3(20)
      REAL MACLA(20), MACLO(20)

      RAD = 57.29577951
      DO 10 I=1,NBAS
      XP1(I) = XP1(I)/RAD
      XP2(I) = XP2(I)/RAD
   10 CONTINUE
      DO 20 I=1,NSTAGE
      DO 30 J=1,NBAS
      X1(I,J) = 3437.74677*ACOS(SIN(MACLA(I))*SIN(XP1(J))+
     1 COS(MACLA(I))*COS(XP1(J))*COS(ABS(XP2(J)-MACLO(I))))
      IF (J.EQ.1) THEN
      X2(I) = X1(I,J)
      X3(I) = J
      GO TO 30
      ENDIF
      IF (X1(I,J).LT.X2(I)) THEN
      X2(I) = X1(I,J)
      X3(I) = J
      ENDIF
   30 CONTINUE
   20 CONTINUE
      RETURN
      END


      SUBROUTINE DISXRT (MACLA,MACLO,XRT,NSTAGE)

      COMPUTES GREAT CIRCLE DISTANCE/AIRLIFTER ROUTE (NO. HEMISPHERE)

      REAL MACLA(20), MACLO(20), XRT(20)

      RAD = 57.29577951
      DO 1  I=1,NSTAGE
      MACLA(I) = MACLA(I)/RAD
      MACLO(I) = MACLO(I)/RAD
    1 CONTINUE
      L = NSTAGE - 1
      DO 2  I=1,L
      XRT(I) = 3437.74677*ACOS(SIN(MACLA(I))*SIN(MACLA(I+1))+
     1 COS(MACLA(I))*COS(MACLA(I+1))*COS(ABS(MACLO(I+1)-MACLO(I))))
    2 CONTINUE
      RETURN
      END
```

Appendix D

"Complex" Method of Box

The Constrained Simplex (Complex) method, as modified for the AR optimization problem, searches for the minimum value of the return function $r(X_i,S_i)$ subject to constraints of the form:

$$G_i \leq X_i \leq H_i \tag{26}$$

where the subscript i refers to the stage number presently being minimized. Only explicit constraints are applicable for the AR optimization program; however, the algorithm also handles implicit constraints on the decision variables if desired. The Complex program actually maximizes the negative of the return function in searching for a minimum. It also assumes that an initial point $X_1,\ldots,X_n$ is available which satisfies all stage constraints; for this application, a decision value of zero is used for the initial point.

The algorithm proceeds as follows:

1. An original "complex" of K=3 points is generated, consisting of the initial feasible starting point and K-1 additional points. These points are obtained by using a pseudo-random number generator and the ranges of the independent decision variable, such that

$$X_i = G_i + R_i(H_i-G_i) \tag{27}$$

where R_i is a pseudo-random deviate uniformly distributed over the interval (0,1).

2. The selected points must satisfy both explicit and implicit constraints. If a point does not satisfy the explicit constraints, it is moved to a value of $\delta = 1 \times 10^{-6}$ inside the appropriate boundary. If an implicit constraint is violated, the point is moved half-way towards the centroid of the other two points, such that

$$X_{i,j} = (X_{i,j}(\text{old}) + \overline{X}_{i,c})/2 \tag{28}$$

where the coordinates of the centroid of the remaining points, $\overline{X}_{i,c}$, are defined by

$$X_{i,c} = \frac{1}{K-1} \sum_{j=1}^{K} [X_{i,j} - X_{i,j}(\text{old})] \tag{29}$$

This is done until the implicit constraints are satisfied.

3. The objective function is evaluated at each point. The point with the lowest function value is moved to a distance $\alpha = 1.3$ times as far as the centroid of the remaining two points as the distance of the rejected point. The new position is on the line joining the rejected point and the centroid, thus:

$$X_{i,j}(\text{new}) = (\overline{X}_{i,c} - X_{i,j}(\text{old})) + \overline{X}_{i,c}. \tag{30}$$

(Ref 16:371)

4. If a point repeats as the lowest function value, it is moved one-half the distance to the centroid of the remaining points. If at any time the new point violates one of the explicit or implicit constraints, its position is adjusted as discussed earlier.

5. The process continues until the objective function values for all three points remain within $\beta = .01$ for $\gamma = 5$ consecutive iterations. An iteration is defined as the calculations required to select a new point which satisfies the constraints and does not repeat in yielding the lowest function value (Ref 16:368). The maximum number of iterations allowed during each optimization is defined by the parameter ITMAX.

Figure D-1 contains a logic flow diagram illustrating the above procedure, as adapted from Reference 16.

Subroutine COMPLX

Subroutine COMPLX can be thought of as the main line program for the optimization portion of the AR optimization model. It coordinates the special purpose and general subroutines plus prints intermediate optimal solutions if the user desires. Subroutines CHECK and CENTR are the general subroutines used exclusively with COMPLX during the optimization process; FUNC and CONST are special purpose subroutines which are also used by DYNAM and other subroutines (see Appendix B).

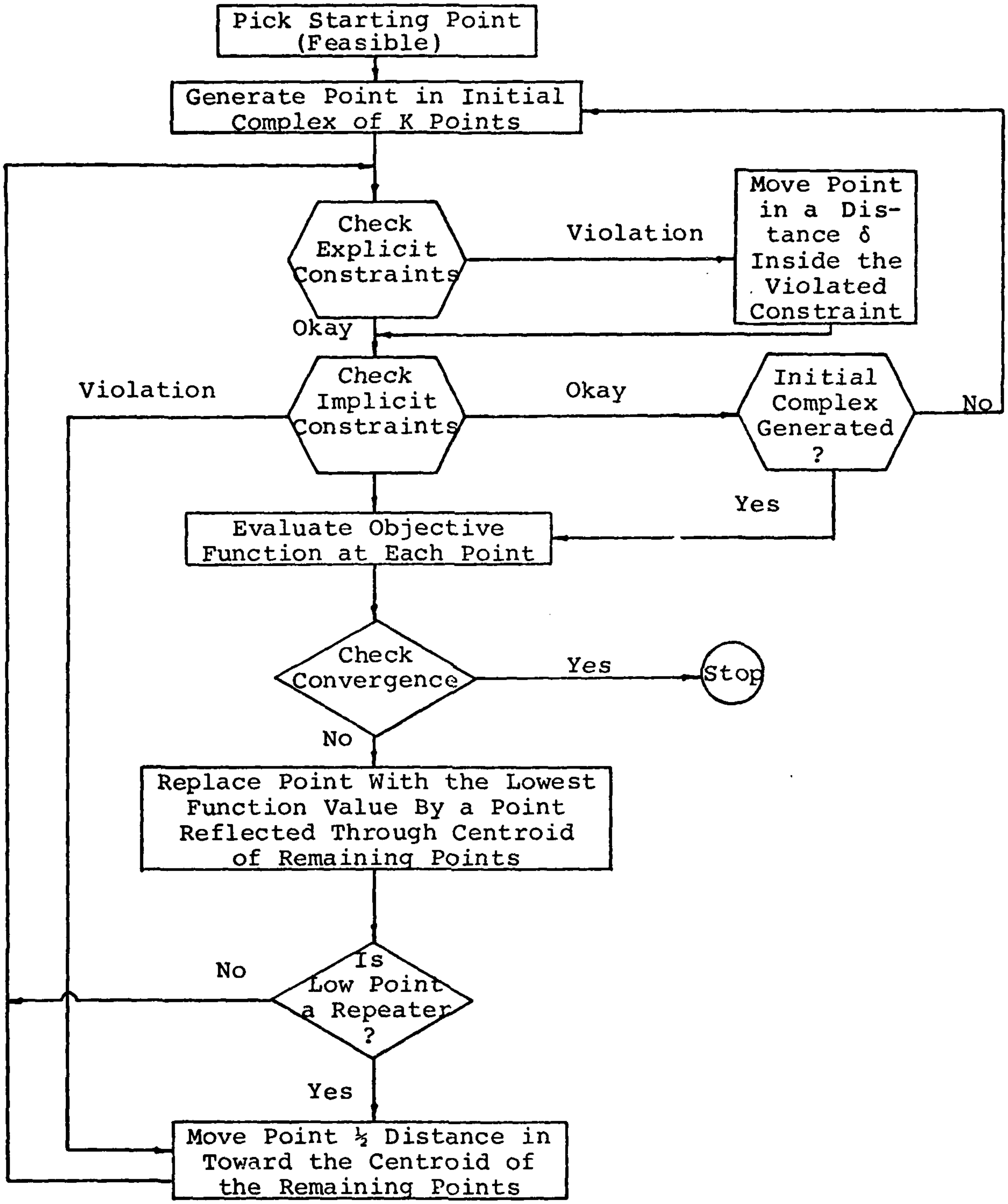


Fig. D-1. Box ("COMPLEX" ALGORITHM) Logic Diagram

The model parameters used in COMPLX are as recommended by Box in Reference 6. These parameters and their values are:

1. α (ALPHA) = 1.3
2. β (BETA) = .01
3. ITMAX = 150
4. γ (GAMMA) = 5
5. K = 3
6. δ (DELTA) = 1×10^{-6}

Subroutine CHECK

CHECK uses subroutine CONST to insure all points in the "complex" meet all explicit and implicit constraints. If a constraint is violated, CHECK applies a correction to bring the point back within the constraint, as previously discussed.

Subroutine CENTR

Subroutine CENTR is used exclusively to calculate the centroid of the vertices. It is called by COMPLX and CHECK whenever a centroid is required.

Appendix E

Sample Output

```
SYSTEM OPTIMUM RETURN =    2.38358[illegible]2E+ 5

          MAXIMUM STAGE RETURNS

STAGE  1 RETURN =    0.

STAGE  2 RETURN =    3.393[illegible]6435E+04

STAGE  3 RETURN =    3.73393[illegible]2[illegible]E+04

STAGE  4 RETURN =    2.35[illegible]8[illegible]8E+ 4

STAGE  5 RETURN =    2.817[illegible]E+04

STAGE  6 RETURN =    2.[illegible]3678335E+ 4

STAGE  7 RETURN =    2.[illegible]3[illegible]7[illegible]8E+ 4

STAGE  8 RETURN =    2.7[illegible]674[illegible]E+ 4

STAGE  9 RETURN =    2.29[illegible]2395E+04
```

Fig. E-1. Sample Raw Data Output
(Dover AFB--Moskow)

```
OPTIMUM DECISIONS

STAGE 1        X(1,1) =   1.[illegible]E-16
STAGE 2        X(2,1) =   1.[illegible]E-16
STAGE 3        X(3,1) =   7.[illegible]73[illegible]E-11
STAGE 4        X(4,1) =   1.[illegible]E-16
STAGE 5        X(5,1) =   1.[illegible]E-16
STAGE 6        X(6,1) =   1.[illegible]E-16
STAGE 7        X(7,1) =   1.[illegible]E-16
STAGE 8        X(8,1) =   1.[illegible]E-16
STAGE 9        X(9,1) =   1.[illegible]E-16

THE SYSTEM OUTPUT STATE =   7.593[illegible]E+[illegible]4
THE INPUT STATE TO STAGE  1 =   7.5930[illegible]81[illegible]E+04
THE INPUT STATE TO STAGE  2 =   1.[illegible]866[illegible]2[illegible]E+05
THE INPUT STATE TO STAGE  3 =   [illegible].1561136[illegible]E+04
THE INPUT STATE TO STAGE  4 =   6.[illegible]353[illegible]84[illegible]E+04
THE INPUT STATE TO STAGE  5 =   5.21445[illegible]2[illegible]E+04
THE INPUT STATE TO STAGE  6 =   1.1172[illegible]472E+05
THE INPUT STATE TO STAGE  7 =   1.3[illegible]691135E+05
THE INPUT STATE TO STAGE  8 =   1.[illegible]17827[illegible]E+05
THE SYSTEM INPUT STATE =   1.8[illegible]E+05
```

Fig. E-1--Continued

AERIAL REFUELING OPTIMIZATION PROGRAM

SUMMARY OF RESULTS

AIRLIFTER ROUTE OF FLIGHT:

STAGE	COORDINATES		WIND FACTOR	DISTANCE (NM)
1	51.99	-37.42	31.	686.73
2	[illegible]	-18.82	42.	[illegible]3.63
3	54.45	-2.32	52.	531.13
4	56.[illegible]	17.[illegible]	57.	617.24
5	53.[illegible]	3[illegible]	59.	435.4[illegible]
6	49.3[illegible]	[illegible]	6[illegible].	519.52
7	[illegible]	[illegible]	67.	552.33
8	41.6[illegible]	62.27	77.	464.73
9	[illegible]	72.46	[illegible]	[illegible]

Fig. E-2. Sample Formated Data Output
(Dover AFB--Moscow)

TANKER DEPARTURE BASES:

BASE NO.	COORDINATES	
1	52.23	-.73
2	47.[illegible]3	68.36

AIRLIFTER DIVERSION BASES

BASE NO.	COORDINATES	
1	4[illegible].[illegible]	72.[illegible]5
2	3[illegible].[illegible]8	-37.[illegible]2
3	52.23	-.73

TYPE AIRLIFTER:	C-5A
CARGO LOAD:	174[illegible]. LBS.
TAKEOFF FUEL:	184[illegible]. LBS.
REQD OVERHEAD DESTINATION FUEL:	754[illegible]. LBS.

TYPE TANKER: KC-135A

Fig. E-2--Continued

STAGE	NO. TANKERS	OFFLOAD (LBS)	TOTAL FUEL CONSUMED (LBS)	DIVERSION BASE	TANKER BASE	FUEL STATE
1	.[illegible]	[illegible].	0.	2	1	75936.
2	.3[illegible]	0.	33934.	3	1	1[illegible]864.
3	.71	923[illegible]1.	57389.	3	1	41561.
4	.[illegible]	[illegible].	235[illegible]1.	3	1	6[illegible]354.
5	.[illegible]	[illegible].	281[illegible]1.	3	1	9[illegible]145.
6	.[illegible]	0.	2[illegible]367.	1	2	111726.
7	.[illegible]	[illegible].	24[illegible]8.	1	2	135691.
8	.[illegible]	[illegible].	27[illegible]5.	1	2	161783.
9	.[illegible]	[illegible].	229[illegible]5.	1	2	184[illegible].

OPTIMUM TOTAL FUEL CONSUMED: 236356. .35.

Fig. E-2--Continued

Appendix F

Sample Problem

This appendix contains a sample problem used to validate the general DYNAM program before it was modified for the AR optimization model. The problem was taken from Reference 14 and formulated in Reference 16. It was solved on the CDC 6600 computer, using seven seconds of CPU time.

The problem:

$$\text{Maximize } F = \sum_{i=1}^{3} r_i$$

Return Function	$r_i = 5.0d_i - id_i$
Transition Function	$S_i = S_i - .4d_i$
Constraint	$0 \leq d_i \leq S_i$
State Variable Constraints	$0 \leq S_i \leq 5$
Starting Point	$d_i = 1.0$
System Input	$S_n = 5.0$
System Output	$S_o = 0.0$
Parameters	NSTAGE = 3
	IPROB = 1
	ALPHA = 1.3
	BETA = .01
	GAMMA = 5
	DELTA = $1\text{x}10^{-6}$
	K = 3
	IPRINT = 0

$$ITMAX = 500$$

$$IOPT = +1$$

$$NSTEP = 100$$

Answers:

$$F = 11.43$$

$$d_1 = 2.447$$

$$d_2 = 1.190$$

$$d_3 = .747$$

Figure F-1 contains the final output for this problem. Algorithm answers were totally correct.

```
              DYNAMIC PROGRAMMING PROCEDURE

      OPTIMIZATION OF  3 STAGE INITIAL VALUE PROBLEM


      SYSTEM OPTIMUM RETURN =    1.14[illegible]983E+01

            MAXIMUM STAGE RETURNS

      STAGE  1 RETURN =   6.191[illegible]74[illegible]8E+[illegible]

      STAGE  2 RETURN =   3.1[illegible]12[illegible]532E+[illegible]

      STAGE  3 RETURN =   2.[illegible]8257962E+[illegible]

            OPTIMUM DECISIONS

      STAGE  1            X(1,1) =   2.41741307E+00

      STAGE  2            X(2,1) =   1.264[illegible]5797E+00

      STAGE  3            X(3,1) =   8.474337[illegible]6E-01

      THE SYSTEM OUTPUT STATE =   3.233[illegible]3613E+[illegible]

      THE INPUT STATE TO STAGE  1 =   [illegible].2[illegible]16[illegible]E+00

      THE INPUT STATE TO STAGE  2 =   [illegible].7[illegible]E+00

      THE SYSTEM INPUT STATE =   5.[illegible]E+00
```

Fig. F-1. Sample Problem Output

Appendix G

Experimental Design Output Data

The output in this appendix is organized into the same four parts used in the Analysis, Chapter IV. Parts One and Two contain the data used in the main experimental design and includes an analysis of variance (ANOVA); Part Three addresses the sensitivity analysis of model parameters, and Part Four contains the data used in verifying the model against actual computer flight plans.

Parts One and Two

Tables G-1 through G-3 contain the output data from the main experimental design found in Table 5. Included are the minimum total fuel consumed figures, total offload, and the number of tankers used for each factor level. The fuel figures are plotted against takeoff fuel and cargo load in Figures 9-14 in Chapter IV.

Analysis of Variance (ANOVA)

An analysis of variance (ANOVA) was performed using the Statistical Package for the Social Sciences (SPSS) ANOVA routine. Only the C-5A total fuel consumed data were used in the analysis (to insure two observations per cell) with independent variables TOFUEL (airlifter takeoff fuel) and LOAD (airlifter cargo load). The objective was to determine which of these two factors had the most effect on total fuel consumed. Figure G-1 shows the resulting SPSS ANOVA table.

TABLE G-1

EXPERIMENTAL DESIGN OUTPUT DATA
C-5A/KC-135A

Load (1000 lb)	T.O. Fuel (1000 lb)	Fuel Offload	Fuel Consumed	# Tankers
100.0	46.0	310565	474690	4
	71.0	278417	424520	3
	96.0	255223	426395	3
	121.0	219071	423080	2
	146.0	205584	412969	2
	171.0	183468	404135	2
	196.0	157403	398837	2
125.0	46.0	309275	448497	3
	71.0	290502	436669	3
	96.0	269400	439602	3
	121.0	243354	438611	3
	146.0	222904	441634	3
	171.0	197904	441634	3
	196.0	172904	441634	3
150.0	46.0	331030	451123	3
	71.0	309174	454324	3
	96.0	285666	454589	3
	121.0	255035	449919	3
	146.0	233174	454324	3
	171.0	206343	424239	2
	196.0	184778	425724	2

TABLE G-2

EXPERIMENTAL DESIGN OUTPUT DATA
C-5A/KC-10A

Load (1000 lb)	T.O. Fuel (1000 lb)	Fuel Offload	Fuel Consumed	# Tankers
100.0	46.0	331206	512939	3
	71.0	305407	512797	3
	96.0	280407	512797	3
	121.0	255407	512797	3
	146.0	230407	512797	3
	171.0	205407	512797	3
	196.0	181206	512939	3
125.0	46.0	319385	540821	3
	71.0	295312	514078	3
	96.0	270959	467583	2
	121.0	252532	474171	2
	146.0	238364	478258	2
	171.0	207599	525368	2
	196.0	183189	528185	2
150.0	46.0	361522	626615	3
	71.0	336522	626615	3
	96.0	311522	626615	3
	121.0	286522	626615	3
	146.0	261522	626615	3
	171.0	230956	555899	2
	196.0	202146	539997	2

TABLE G-3

EXPERIMENTAL DESIGN OUTPUT DATA
C-141B/KC-135A

Load (1000 lb)	T.O. Fuel (1000 lb)	Fuel Offload	Fuel Consumed	# Tankers
20.0	30.0	206594	247578	2
	45.0	191594	247578	2
	60.0	176594	247578	2
	75.0	109869	234235	2
	90.0	146594	247578	2
	105.0	131594	247578	2
	120.0	116594	247578	2
35.0	30.0	170660	289035	3
	45.0	155660	289035	3
	60.0	140660	289035	3
	75.0	125660	289035	3
	90.0	110660	289035	3
	105.0	95660	289035	3
	120.0	80660	289035	3
50.0	30.0	165125	316393	4
	45.0	156953	290356	3
	60.0	131908	286195	3
	75.0	116817	253858	2
	90.0	107432	257652	2
	105.0	90744	286581	3
	120.0	82502	261625	2

Source of Variation	Sum of Squares	DF	Mean Square		Signif of F
Main Effects	.32184E+11	8	.402E+10	.618	.752
TOFUEL	.47147E+10	6	.785E+09	.121	.993
LOAD	.27469E+11	2	.137E+11	2.110	.146
2-Way Interactions	.84180E+10	12	.701E+09	.108	.999
TOFUEL LOAD	.84180E+10	12	.701E+09	.108	.999
Explained	.40602E+11	20	.203E+10	.312	.994
Residual	.13667E+12	21	.650E+10		
Total	.17727E+12	41	.432E+10		

Fig. G-1. SPSS ANOVA Table

The analysis of variance assumes the following model:

$$y_{ij} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ij} \tag{31}$$

where

y_{ij} = calculated total fuel consumed from model output

μ = overall mean of the total fuel consumption figures

α_i = the effect of cargo load on total fuel consumed

β_j = the effect of takeoff fuel on total fuel consumed

$(\alpha\beta)_{ij}$ = the effect of interaction between the two factors on total fuel consumed

ε_{ij} = error term

Using a significance level of .95, the SPSS output indicates that none of the factors, including interaction, are statistically significant in explaining the model. However, cargo load does have a much greater effect on total fuel consumed than the takeoff fuel, with a significance of .854 versus a negligible .007 for the takeoff fuel factor.

Therefore, the null hypotheses

$$H_o: \quad \alpha_1 = \alpha_2 = \alpha_3 \tag{32}$$

$$H_o: \quad \beta_1 = \beta_2 = \ldots = \beta_7 \tag{33}$$

$$H_o: \quad (\alpha\beta)_{11} = (\alpha\beta)_{12} = \ldots = (\alpha\beta)_{ij} \qquad i=1,2,3 \quad j=1,2,\ldots,7 \tag{34}$$

cannot be rejected at the significance level of .95; hypothesis (32) becomes significant at a level of .85, but the others are insignificant. This confirms that cargo load has a much greater effect than takeoff fuel on total fuel consumed.

Part Three

A C-5A mission from Rhein Main AB, West Germany, was selected to study the effect of the parameters NSTEP and ITMAX on the model output. This was done to determine a value for each of these parameters that would minimize computer execution time, since they have the most effect on the total number of computations. Also, these results were

used with other output during the validation analysis presented in Chapter IV. Table G-4 contains the data used in this analysis.

Other model parameters were not changed from the values recommended by Box (Ref 6). These recommended values were the result of a sensitivity analysis presented in Reference 6, and insure the most efficient application of the Box "Complex" algorithm.

Part Four

Six typical aerial refueling missions were selected for comparison between model output and actual computer flight plan data. Wind factors, route of flight, tanker bases, and diversion bases were obtained from actual computer flight plans and used as input to the AR optimization model. Since the computer flight plan data were not based on optimization techniques, the comparison shows the AR optimization model's feasibility and also serves to validate model output. Figures G-3 through G-8 compare the model output for the following airlifter missions:

1. Rhein Main AB, West Germany - Dover AFB, DE, Figure G-3 (KC-135A/C-5A). A takeoff fuel of 100,000 pounds was used with the model, versus 225,200 pounds with the computer flight plan. This somewhat distorts the figures for comparison, but dramatizes the effect of excessive fuel loads on overall fuel consumption. The

AIRLIFTER ROUTE OF FLIGHT:

STAGE	COORDINATES		WIND FACTOR	DISTANCE (NM)
1	39.11	75.46	-8	1005.44
2	54.81	66.75	-32	764.30
3	60.00	45.00	-9	732.29
4	62.00	20.00	-36	783.38
5	54.11	.13	-18	382.56
6	50.25	-8.16	0	0.00

TANKER DEPARTURE BASES:

COORDINATES		BASE
52.23	-.73	Mildenhall, England
47.43	68.36	Loring AFB, ME

AIRLIFTER DIVERSION BASES:

COORDINATES		BASE
50.25	-8.16	Rhein Main, West Germany
39.11	75.46	Dover AFB, DE
53.31	60.55	Goose Bay, Labrador

Fig. G-2. Input Data for Part Three

TABLE G-4

NSTEP AND ITMAX EFFECT ON OUTPUT

NSTEP	ITMAX	TOTAL FUEL CONSUMED	OFFLOAD
25	150	231140	105684
50	150	228653	103817
75	150	228748	107073
100	150	229246	111154
100	50	229246	111154
100	100	229246	111154
100	200	229246	111154
100	500	229246	111154

computer flight plan shows a refueling point only 382 nautical miles from that selected by the model. Computer flight plan data results in a significantly higher fuel state overhead destination than required, also adding to airlifter fuel consumption.

2. Tinker AFB, OK - Kadena AB, Japan, Figure G-4 (KC-135A/C-5A). Again, the computer flight plan calls for an onload much greater than that required, and a significant portion of this extra fuel is burned just by carrying it. Fuel state values are in good agreement. The AR point chosen was exactly the same as that used in the computer flight plan. Again, fuel overhead destination for the computer flight plan is much too high.

3. Dover AFB - Moskow, Russia, Figure G-5 (KC-135A/C-5A). The model calls for one AR at stage 3 as compared to two ARs at stages 4 and 8 for the computer flight plan. From the model results, it appears that the first AR was not necessary. Again, too much fuel offload and fuel overhead destination resulted in a higher fuel consumed figure for the computer flight plan.

4. Dover AFB, DE - Cairo, Egypt, Figure G-6 (KC-135A/C-5A). Again, the first AR shown on the computer flight plan was not necessary. This results in the airlifter carrying an excessive amount of fuel over a large portion of the flight. This results in a higher fuel consumption and excessive fuel overhead destination. AR points and fuel states are in good agreement.

5. Tinker AFB, OK - Hickam AFB, HI, Figure G-7 (KC-135A/C-5A). This example was included to illustrate the strong agreement between computer flight plan fuel states and those computed from the model. The large discrepancy (about 6000 pounds) at stage 3 was caused by an altitude change in the computer flight plan for a planned AR rendezvous (no fuel offload was planned, however). This resulted in slightly higher fuel consumption. No AR was required here, nor was it called for by the model.

6. McGuire AFB, NJ - Bahrain, Bahrain, Figure G-8 (KC-135A/C-141B). This example shows reasonable agreement

between AR points and fuel states. Again, the computer flight plan calls for excessive fuel offloads, resulting in higher fuel consumption and higher than required fuel overhead destination.

STAGE	COORDINATES		WIND FACTOR	DISTANCE	OFFLOAD	DIV. BASE	TNKR BASE	FUEL STATE	COMPUTER FLT PLAN FUEL STATE	COMPUTER FLT PLAN OFFLOAD
1	39.11	75.46	-8	1005.44	0	2	2	25995	38100	0
2	54.81	66.75	-32	764.30	0	3	2	71880	83200	0
3	60.00	45.00	-9	732.29	0	3	2	110143	130700	0
4	62.00	20.00	-36	783.38	0	1	1	147346	170100	0
5	54.11	.13	-18	382.56	0	1	1	191804	194700	29600
6	50.25	-8.16	0	0	111154	1	1	100000	225200	0

AIRLIFTER DIVERSION BASES:

1	50.25	-8.16	Rhein Main, Germany
2	39.11	75.46	Goose Bay, Labrador
3	53.31	60.55	Dover AFB, DE

TANKER DEPARTURE BASES:

1	52.23	-.73	Mildenhall, England
2	47.43	68.36	Loring AFB, ME

TYPE AIRLIFTER:	C-5A
CARGO LOAD:	100,000 lb
TAKEOFF FUEL:	100,000 lb
REQ OVERHEAD DESTINATION:	25,000 lb
TYPE TANKER:	KC-135
AIRLIFTER FUEL CONSUMED (MODEL)	185,159 lb
AIRLIFTER FUEL CONSUMED (CFP)	219,500 lb

Fig. G-3. Rhein Main - Dover AFB

STAGE	COORDINATES		WIND FACTOR	DISTANCE	OFFLOAD	DIV. BASE	TNKR BASE	FUEL STATE	COMPUTER FLT PLAN FUEL STATE	COMPUTER FLT PLAN OFFLOAD
1	26.35	-127.76	-8	933.48	0	1	2	19197	60000	0
2	37.15	-140.98	-27	580.61	0	1	2	57688	102500	0
3	44.0	-150.0	-26	509.37	0	1	2	81681	128100	0
4	49.33	-159.65	-49	460.41	0	2	1	104906	151000	0
5	53.5	-170.0	-28	740.01	0	2	1	127430	173900	0
6	59.0	170.0	-35	791.26	0	2	1	162738	212200	0
7	56.16	145.83	-25	697.59	17159	2	1	186480	182400	77200
8	50.78	128.43	-21	610.05	0	4	1	223627	221300	0
9	44.76	116.2	-21	836.61	0	4	1	258189	255900	0
10	37.0	100.82	0	0	0	3	1	308000	308000	0

AIRLIFTER DIVERSION BASES:

1	26.35	-127.76	Kadena, Japan
2	61.25	149.8	Elmendorf AFB, AL
3	37.0	100.82	Tinker AFB, OK
4	47.43	122.32	Travis AFB, CA

TANKER DEPARTURE BASES:

1	61.25	149.8	Elmendorf AFB, AL
2	26.35	-127.76	Kadena, Japan

TYPE AIRLIFTER:	C-5A
CARGO LOAD:	54,000 lb
TAKEOFF FUEL:	308,000 lb
REQ OVERHEAD DESTINATION:	18,500 lb
TYPE TANKER:	KC-135A
AIRLIFTER FUEL CONSUMED (MODEL)	305962 lb
AIRLIFTER FUEL CONSUMED (CFP)	344900 lb

Fig. G-4. Tinker AFB - Kadena AB

STAGE	COORDINATES		WIND FACTOR	DISTANCE	OFFLOAD	DIV. BASE	TNKR BASE	FUEL STATE	COMPUTER FLT PLAN FUEL STATE	COMPUTER FLT PLAN OFFLOAD
1	55.98	-37.42	31	688.78	0	2	1	75930	102900	0
2	56.0	-16.82	42	503.69	0	3	1	109864	155100	0
3	54.46	-2.32	52	531.13	92301	3	1	41561	166400	0
4	56.0	13.0	57	617.24	0	3	1	64354	118200	81000
5	53.0	30.0	59	436.4	0	3	1	92145	148500	0
6	49.3	40.0	60	519.62	0	1	2	111726	169900	0
7	44.0	50.0	67	552.93	0	1	2	135691	195700	0
8	41.8	62.23	77	464.73	0	1	2	161783	160200	68100
9	40.8	72.46	0	0	0	1	2	184000	184000	0

AIRLIFTER DIVERSION BASES:

1	40.8	72.46	Dover, DE
2	55.98	-37.42	Moscow, Russia
3	52.23	-.73	Mildenhall, England

TANKER DEPARTURE BASES:

1	52.23	-.73	Mildenhall, England
2	47.43	68.36	Loring AFB, ME

TYPE AIRLIFTER:	C-5A
CARGO LOAD:	174,000 lb
TAKEOFF FUEL:	184,000 lb
REQ OVERHEAD DESTINATION:	75,400 lb
TYPE TANKER:	KC-135A
AIRLIFTER FUEL CONSUMED (MODEL)	205,995 lb
AIRLIFTER FUEL CONSUMED (CFP)	230,200 lb

Fig. G-5. Dover AFB - Moskow

STAGE	COORDINATES		WIND FACTOR	DISTANCE	OFFLOAD	DIV. BASE	TNKR BASE	FUEL STATE	COMPUTER FLT PLAN FUEL STATE	COMPUTER FLT PLAN OFFLOAD
1	30.13	-31.4	53	623.92	0	2	1	91102	125000	0
2	36.3	-21.38	36	577.05	0	2	1	119961	159400	0
3	42.03	-11.4	47	676.92	0	3	1	148473	189000	0
4	46.16	3.25	52	501.64	109874	3	1	71675	112300	120200
5	47.8	15.28	63	598.83	0	3	1	92838	138400	0
6	47.0	30.0	65	417.05	0	3	1	119680	167400	0
7	46.0	40.0	69	440.57	0	1	3	138003	188700	0
8	44.0	50.0	57	552.93	0	1	3	158572	211300	0
9	41.8	62.23	64	450.23	0	1	3	185425	183000	63700
10	40.86	72.15	0	0	0	1	3	208000	208000	0

AIRLIFTER DIVERSION BASES:

1	39.16	75.5	Dover AFB, DE
2	30.13	-31.4	Cairo, Egypt
3	40.46	3.58	Torrejon AB, Spain

TANKER DEPARTURE BASES:

1	40.46	3.58	Torrejon, AB, Spain
2	52.23	-.73	Mildenhall, England
3	47.43	68.36	Loring AFB, ME

TYPE AIRLIFTER:	C-5A
CARGO LOAD:	150,000 lb
TAKEOFF FUEL:	208,000 lb
REQ OVERHEAD DESTINATION:	90,200 lb
TYPE TANKER:	KC-135A
AIRLIFTER FUEL CONSUMED (MODEL)	231,747 lb
AIRLIFTER FUEL CONSUMED (CFP)	266,900 lb

Fig. G-6. Dover AFB - Cairo, Egypt

STAGE	COORDINATES		WIND FACTOR	DISTANCE	OFFLOAD	DIV. BASE	TNKR BASE	FUEL STATE	COMPUTER FLT PLAN FUEL STATE	COMPUTER FLT PLAN OFFLOAD
1	21.26	157.7	-96	184.01	0	3	1	54648	50300	0
2	24.05	156.32	-106	647.69	0	3	1	64232	58400	0
3	29.93	146.15	-29	599.2	0	3	1	98196	92100	0
4	34.6	135.7	-18	653.66	0	1	1	125942	122600	0
5	39.05	123.26	-22	521.78	0	1	1	157220	155600	0
6	37.68	112.3	-38	532.84	0	1	1	183802	182600	0
7	35.35	101.63	0	0	0	2	2	211000	211000	0

AIRLIFTER DIVERSION BASES:

1	38.26	121.93	Travis AFB, CA
2	35.35	101.63	Tinker AFB, OK
3	21.26	157.7	Hickam AFB, HI

TANKER DEPARTURE BASES:

1	38.26	121.93	Travis AFB, CA
2	34.63	99.63	Altus AFB, OK

TYPE AIRLIFTER:	C-5A
CARGO LOAD:	54,000 lb
TAKEOFF FUEL:	211,000 lb
REQ OVERHEAD DESTINATION:	17,900 lb
TYPE TANKER:	KC-135A
AIRLIFTER FUEL CONSUMED (MODEL)	156,352 lb
AIRLIFTER FUEL CONSUMED (CFP)	160,700 lb

Fig. G-7. Tinker AFB - Hickam AFB

STAGE	COORDINATES		WIND FACTOR	DISTANCE	OFFLOAD	DIV. BASE	TNKR BASE	FUEL STATE	COMPUTER FLT PLAN FUEL STATE	COMPUTER FLT PLAN OFFLOAD
1	26.26	-50.15	39	937.56	0	1	2	22804	43100	0
2	25.75	-32.76	74	733.97	0	4	1	44721	67100	0
3	34.61	-22.98	61	800.72	0	4	1	60687	90000	0
4	40.8	-8.0	41	674.82	0	3	1	80248	76800	38600
5	46.55	5.4	46	599.75	23358	3	1	74283	97200	0
6	48.0	20.0	71	403.61	0	5	2	89686	56200	61100
7	47.58	30.0	96	421.28	25416	5	2	74212	68100	0
8	46.0	40.0	118	440.57	0	2	1	83910	80000	0
9	44.0	50.0	133	552.93	19897	2	1	73306	92200	0
10	41.8	62.23	88	474.94	0	2	2	84739	82900	24500
11	40.58	72.63	0	0	0	2	2	96200	96200	0

AIRLIFTER DIVERSION BASES:

1	26.26	-50.15	Bahrain
2	39.16	75.5	Dover AFB, DE
3	40.46	3.58	Torrejon, Spain
4	38.0	-23.63	Athens, Greece
5	52.23	-.73	Mildenhall, England

TANKER DEPARTURE BASES:

1	40.46	3.58	Torrejon, Spain
2	52.23	-.73	Mildenhall, England

TYPE AIRLIFTER:	C-141B
CARGO LOAD:	68,000 lb
TAKEOFF FUEL:	96,200 lb
REQ OVERHEAD DESTINATION:	22,300 lb
TYPE TANKER:	KC-135A
AIRLIFTER FUEL CONSUMED (MODEL)	142,067 lb
AIRLIFTER FUEL CONSUMED (CFP)	184,400 lb

Fig. G-8. McGuire AFB - Bahrain

Appendix H

Glossary

ARCP -- The Aerial Refueling Control Point is the point where the tanker orbits while waiting for the airlifter, usually 100 nautical miles downstream from the ARIP. The tanker departs the orbit at the ARCP only when positive radio contact is established with the airlifter and the airlifter is within 80 nautical miles of the tanker. The ARCP also defines the point where the two aircraft ideally complete their rendezvous and begin aerial refueling.

ARIP -- The Aerial Refueling Initial Point is the first point defining the aerial refueling track. The airlifter must have positive radio contact with the tanker before leaving this point. Also, at the ARIP the airlifter begins a descent to 1000 feet below AR altitude and slows to rendezvous airspeed in preparation for the rendezvous.

"BUDDY" AERIAL REFUELING -- This mode of aerial refueling occurs when the tanker and airlifter takeoff in a cell formation, and the tanker accompanies the airlifter along its route of flight. The tanker offloads its fuel as required by the airlifter. The tanker can then either accompany the airlifter for another AR, or return to a recovery base.

CATEGORY I ROUTE -- Simply, this refers to the overwater portion of an airlift route segment. By definition, the Category I portion of a route is that segment where ground based radio aids to navigation are inadequate to accurately position the aircraft at least once each hour. The definition requires the aircraft to fly directly over the navigation aid to be "accurately positioned."

END-AR POINT -- This point defines the end of the aerial refueling track; if AR is not complete by this point, special permission from Air Traffic Control is required. At this point, both airlifter and tanker depart the track and continue to their destinations.

GREAT CIRCLE ROUTE -- A great circle route is the shortest distance between two points on the earth's surface. If one were to pass a plane through the center of the earth and intersect two points on the earth's surface, the line defined by the intersection of the earth's surface and the plane would be the great circle route between those two points.

INDICATED AIRSPEED -- This is the airspeed read from the aircraft's airspeed indicator. True airspeed is calculated from indicated by applying corrections for

installation error, air compressibility effects, temperature and altitude.

KNOTS -- Nautical miles per hour.

M-14 -- M-14 is a computer simulation model of the Military Airlift Command's airlift system. The model makes extensive use of probability theory and statistical sampling while simulating the effects of uncertainty on the aircraft network. M-14 models all MAC possessed aircraft, crews, maintenance functions, normal airlift missions, and aerial refueling, to name just a few of its many functions. Reference 17 explains the full capability of M-14 and its limitations.

MULTIPLE TANKER AR -- As used in this study, this term refers to the option of refueling from more than one tanker at each rendezvous point. This is contrasted with the restriction that only one tanker can be accepted at a time; this is a major limitation of previous studies on aerial refueling optimization.

NAUTICAL MILE -- A nautical mile is defined as 1/60 of a degree of latitude. It is 1.15 statute miles, or 6076.115 feet.

NORMAL RENDEZVOUS AERIAL REFUELING -- This mode of aerial refueling occurs when the airlifter and tanker takeoff separately and rendezvous at a predetermined point for refueling. This is by far the most common mode. The tanker returns to a recovery base after AR is complete.

OPERATING WEIGHT -- Aircraft operating weight is defined as the total operational weight of the aircraft minus fuel and cargo load. It includes the airframe weight, weight of all internal stores, oil, oxygen, nitrogen (C-5A only), cargo handling equipment, and aircrew.

OPTIMUM ALTITUDE -- This is a commonly used term to refer to the airlifter's performance altitude ceiling. This altitude is defined as providing a 300 feet per minute rate of climb for the C-5A, or a 400 feet per minute rate of climb for the C-141B. It is not an "optimal" altitude in any sense.

TEMPERATURE DEVIATION -- Standard atmospheric temperature is defined as 15° Centigrade at sea level. To find the standard temperature at any altitude, apply the standard lapse rate of -2° Centigrade per 1000 feet. Temperature deviation from standard is the difference between the actual outside air temperature and the temperature which would exist on a standard day. This figure is used in

lieu of the actual temperature during all mission planning.

TRUE AIRSPEED -- Intuitively, true airspeed is the speed of the aircraft through (or relative to) the air mass. To obtain true airspeed, multiply aircraft indicated Mach number by the local speed of sound. Ground speed is calculated from true airspeed by adding the wind factor.

WIND FACTOR -- Wind factor is defined as the average effective wind an aircraft experiences, based on wind direction and aircraft heading, over a route or route segment. For example, a wind factor of 50 indicates the aircraft experiences an average tailwind of 50 knots over the route segment. A negative wind factor indicates a headwind. The wind factor is added algebraically to the true airspeed to obtain ground speed.

Vita

Tenny Albert Lindholm was born on 16 June 1948 in Ketchikan, Alaska, the son of Henry M. and Beverly M. Lindholm. He graduated from the United States Air Force Academy in June, 1970, with a Bachelor's Degree in Aeronautical Engineering. Upon receiving his Commission in the United States Air Force, he attended Undergraduate Pilot Training at Webb AFB, TX. From 1971 through 1976, he served as squadron pilot and instructor pilot with the 3rd Military Airlift Squadron, Dover AFB, DE, flying the C-5A Galaxy. In 1974, he earned a Master's Degree in Business Administration from Southern Illinois University, Edwardsville, Illinois. Prior to entering the School of Engineering at the Air Force Institute of Technology, he served with the 56th Military Airlift Squadron and the 443rd Technical Training Squadron at Altus AFB, OK, where he was a C-5A Flight Examiner Aircraft Commander and the Chief of Aircraft Commander Training, Military Airlift Command. He has over 3000 hours in the C-5A aircraft.

He is married to the former Lenita Kay Hauber of Westminster, Colorado. They have no children.

Permanent Address: 14611 S. E. 45th
Bellevue, Washington 98006

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFIT/GST/OS/82M-7	2. GOVT ACCESSION NO. AD-A115699	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) AN AERIAL REFUELING OPTIMIZATION MODEL APPLIED TO STRATEGIC AIRLIFT		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Tenny A. Lindholm		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Institute of Technology (AFIT/EN) Wright-Patterson AFB, Ohio 45433		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE March 1982
		13. NUMBER OF PAGES 166
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

APPROVED FOR PUBLIC RELEASE: IAW AFR 190-17

4 JUN 1982

~~FREDERIC C. LYNCH, Major, USAF~~
~~Director of Public Affairs~~

Lynn Wolaver
LYNN E. WOLAVER
Dean for Research and Professional Development
AIR FORCE INSTITUTE OF TECHNOLOGY (ATC)
WRIGHT-PATTERSON AFB, OH 45433

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

AERIAL REFUELING
STRATEGIC AIRLIFT
DYNAMIC PROGRAMMING
AIR REFUELING OPTIMIZATION

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

With the advent of strategic airlift in-flight refueling, the role of strategic mobility in supporting our national policy has taken on new dimensions. Airlift of important military equipment to any part of the world is no longer dependent on enroute bases for refueling. Also, our response time to any contingency far from home can now be measured in terms of hours instead of days. This increased demand on our limited aerial

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

refueling resources, plus the high cost and doubtful availability of jet fuel, require that any deployment using aerial refueling be done as efficiently as possible.

The main objective of this study is to define and develop an operationally usable optimization model that will, given airlifter route of flight, takeoff fuel and gross weight, enroute wind factors, tanker departure bases, and available enroute diversion bases, determine the aerial refueling points, fuel offload, and tanker departure bases to minimize total fuel consumed by both airlifter and tanker. The key to this study is the requirement that the model be operationally usable. Therefore, the model is designed to allow input of any mission scenario using geographic coordinates.

The various subobjectives presented in this paper are organized with a view towards validating and verifying the model's output. However, some questions are also answered regarding the nature of the aerial refueling optimization problem. An analysis of the experimental design shows that it is extremely difficult if not impossible to develop general "rules of thumb" to minimize total fuel consumed, primarily because of the complex interaction between the control variables within the aerial refueling system. It is found that cargo load, and hence airlifter gross weight, has more effect on total fuel consumed than takeoff fuel. Also, it is found that the objective of minimizing total fuel consumed is not always consistent with that of minimizing the use of tankers.

The model uses the dynamic programming technique during the solution process. In addition, the "Complex" Method of Box is incorporated to optimize, or minimize, stage return functions.

It is anticipated that the use of this model to plan operational aerial refueling missions will result in significant fuel savings to the Air Force.